REVISION PACK S4

CREDIT MATHS

S4 Credit Mathematics

Revision Pack

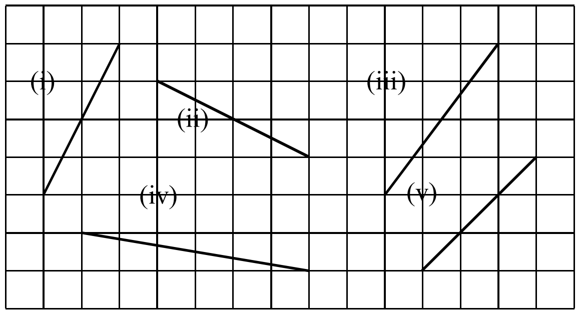
Gradient & Equation of a Line

1. (a) Calculate the gradient of each line in the diagram opposite.

   (b) **Copy** and complete each statement below :

   The gradient of any horizontal line is _______ .

   The gradient of any vertical line is __________ .



2. Calculate the gradient of the line joining each pair of points below.

   (a) A(4,3) , B(8,11) (b) C(1,9) , D(3,1) (c) E(-2,6) , F(8,8)

   (d) G(5,-9) , H(8,-15) (e) I(0,6) , J(5,11) (f) K(-1,-3) , L(7,-9)

   (g) M(-4,0) , N(-1,5) (h) P(2,2) , Q(-3,4) (i) R(5,-1) , S(-2,10)

3. Draw each of the following lines on a coordinate diagram *(you should manage 3 lines on each diagram)*.

   (a) $y = x + 3$ (b) $y = -2x - 1$ (c) $y = \frac{1}{2}x$

   (d) $y = -\frac{1}{2}x + 2$ (e) $x + y = 6$ (f) $2y = x - 4$

   (g) $3y = x + 12$ (h) $4x + 5y = 20$ (i) $3x - 2y = 12$

4. State the gradient and the y-intercept point for each line in question 3.

5. State the gradient and the y-intercept point for each line below.

   (a) $y = x - 7$ (b) $y = -5x + 3$ (c) $5y = 3x - 10$

   (d) $y = -4x$ (e) $2x + y = 11$ (f) $2y = x - 5$

   (g) $3y - x = 18$ (h) $3x + 7y - 21 = 0$ (i) $4x - 5y = 20$

6. Write down the equation of the line:
   (a) with gradient 4, passing through the point (0,5)
   (b) with gradient -2, passing through the point (0,1)
   (c) with gradient $\frac{3}{4}$, passing through the point (0,-3)

7. Establish the equation of the line passing through each pair of points below.

   (a) A(2,1) , B(6,13) (b) C(3,4) , D(5,-4) (c) E(-2,-1) , F(6,3)

   (d) G(4,-13) , H(-2,-1) (e) I(2,8) , J(10,12) (f) K(-3,2) , L(9,-2)

8. *(extension & extra practice)* Find the equation of each line in question 2 using … $y - b = m(x - a)$.

Functions & Graphs (1)

1. A function is given as $f(x) = 6x - 5$.

   Find: (a) $f(3)$ (b) $f(-1)$ (c) $f(\frac{1}{2})$ (d) $f(a)$

2. A function is given as $f(x) = x^2 + 4$.

   Find: (a) $f(2)$ (b) $f(4)$ (c) $f(-3)$ (d) $f(p)$

3. A function is given as $h(a) = 12 - 2a$.

   Find: (a) $h(4)$ (b) $h(6)$ (c) $h(-2)$ (d) $h(m)$

4. A function is defined as $g(x) = x^2 + 3x$.

   Find: (a) $g(a)$ (b) $g(2p)$ (c) $g(m+1)$ (d) $g(2-e)$

5. A function is defined as $f(x) = x^2 - 4x$.

   Find: (a) $f(4)$ (b) $f(3a)$ (c) $f(a-2)$ (d) $f(2p+1)$

6. A function is given as $f(x) = 5x + 3$. For what value of x is :

   (a) $f(x) = 23$ (b) $f(x) = -2$ (c) $f(x) = 5$?

7. A function is given as $h(t) = 20 - 6t$. For what value of t is :

   (a) $h(t) = 2$ (b) $h(t) = -16$ (c) $h(t) = 32$?

8. A function is given as $g(a) = a^2 - 16$. For what value(s) of a is :

   (a) $g(a) = 9$ (b) $g(a) = -15$ (c) $g(a) = 0$?

9. A function is defined as $f(x) = x^2 + 2x$.

   (a) Evaluate: i) $f(3)$ ii) $f(-2)$.

   (b) Find $f(a+3)$ in its simplest form.

10. A function is defined as $h(a) = 33 - 6a$.

    (a) Evaluate: i) $h(4)$ ii) $h(-1)$.

    (b) Given that $h(t) = 0$, find the value of *t*.

    (c) Express $h(p-2)$ in its simplest form.

Functions & Graphs (2)

1. (a) **Copy** and complete the table below for the function with formula $f(x) = x^2 - 2x - 3$.

x	-2	-1	0	1	2	3	4
$f(x)$	5				-3		

(b) Draw the graph the function for $-2 \le x \le 4$ where x is a real number.

(c) Write down the nature and the coordinates of the turning point.

(d) For what value(s) of x is $f(x) = 0$?

2. (a) **Copy** and complete the table below for the function with formula $g(x) = x^2 - 8x + 12$.

x	0	1	2	3	4	5	6	7	8
$g(x)$		5				-3			

(b) Draw the graph the function for $0 \le x \le 8$ where x is a real number.

(c) Write down the nature and the coordinates of the turning point.

(d) For what value(s) of x is $g(x) = 0$?

3. (a) **Copy** and complete the table below for the function with formula $h(x) = 8x - x^2$.

x	-1	0	1	2	3	4	5	6	7	8	9
$h(x)$			7				15				

(b) Draw the graph the function for $-1 \le x \le 9$ where x is a real number.

(c) Write down the nature and the coordinates of the turning point.

(d) For what value(s) of x is $h(x) = 0$?

4. (a) **Copy** and complete the table below for the function with formula $f(x) = 3x^2 - x^3$.

x	-1	0	1	2	3	4
$f(x)$	4					-16

(b) Draw the graph the function for $-1 \le x \le 4$ where x is a real number.

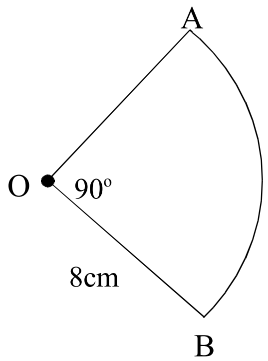
(c) Write down the natures and the coordinates of any turning points.

(d) For what value(s) of x is $f(x) = 0$?

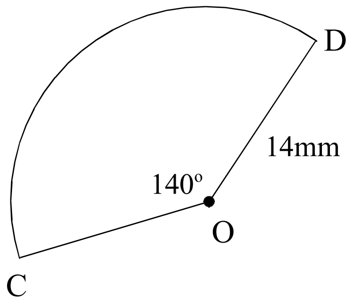
The Circle (1) - Arcs & Sectors

1. Calculate the length of the arc in each diagram below, giving your answer correct to 1d.p.

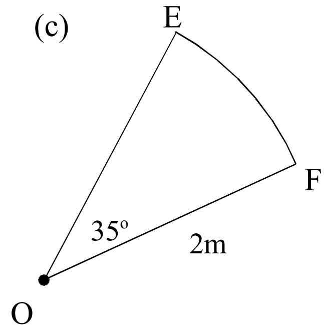
(a)



(b)



(c)



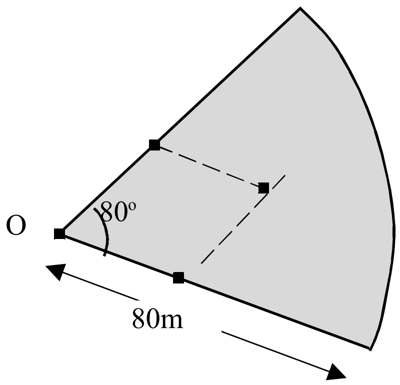
2. Calculate the perimeter of each sector in question1.

3. Calculate the area of each sector in question 1.

4. A school baseball field is in the shape of a sector of a circle as shown. Given that O is the centre of the circle, calculate:

(a) the perimeter of the playing field;

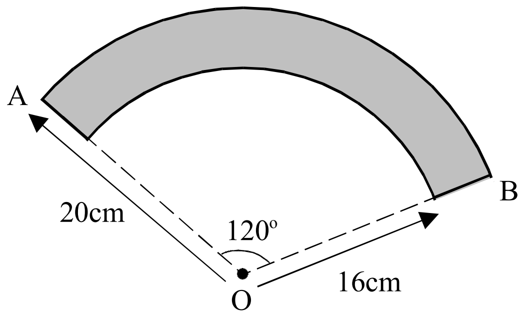
(b) the area of the playing field.



5. In the diagram opposite, O is the centre of two concentric circles with radii 16cm and 20cm as shown. Angle $AOB = 120^o$.

Calculate: (a) The **perimeter** of the shaded shape.

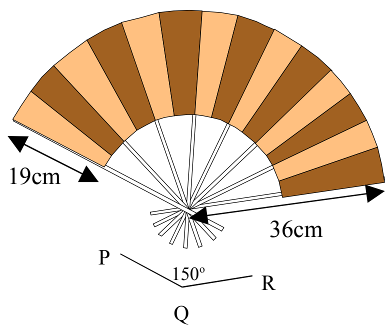
(b) The shaded area.



6. A Japanese paper fan is fully opened when angle $PQR = 150^o$ as shown.

(a) Using the dimensions shown in *diagram 1*, calculate the approximate area of paper material in the fan.

diagram 1



(b) Decorative silk bands are placed along the edges as shown in *diagram 2*, calculate the approximate total

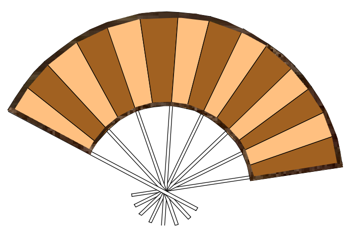


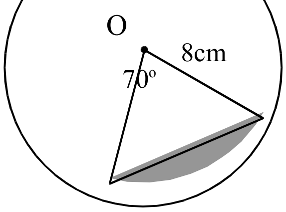
diagram 2

length of this silk edging strip.

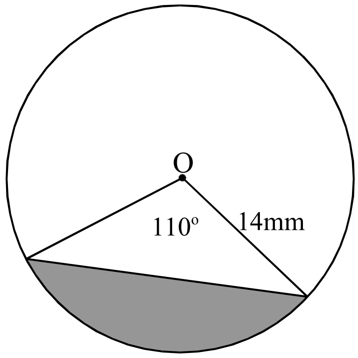
The Circle (2) - Sectors, Segments & Chords

1. Calculate the **area** of each shaded segment in the diagrams below.

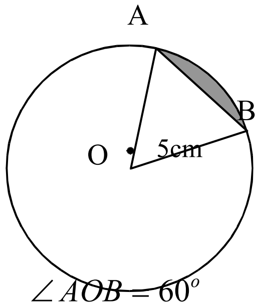
(a)



(b)

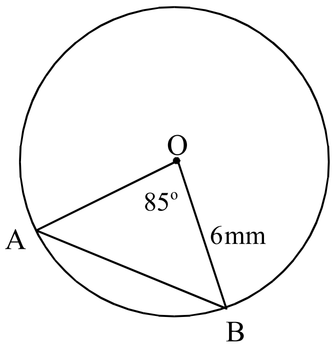


(c)

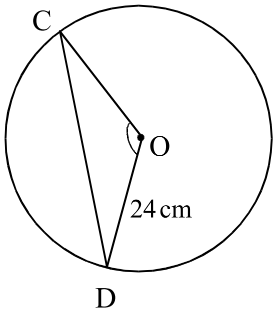


2. Calculate the **lengt**h of the chord in each diagram below.

(a)

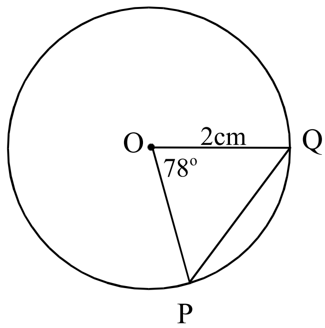


(b)



$\angle COD = 130^o$

(c)

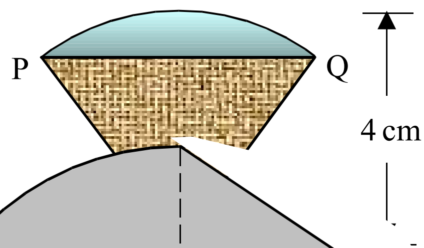


3. Calculate the **perimeter** of each segment in question 2.

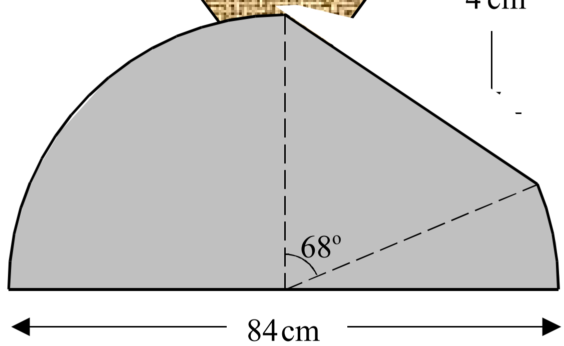
4. The logo for a small ice-cream company is shown opposite. It is simply a sector of a circle with its centre at O.

(a) Calculate the perimeter of the logo.

(b) Calculate the area of the ice-cream part.



5. A designer table-top is shaped as a semi-circle with a segment removed as shown in the diagram.



From the information supplied calculate:

(a) the perimeter of the table-top;

(b) the area of the table-top.

The Circle (3) - Extended Questions

1. (a) A wafer biscuit consists of a sector of a circle with a triangular part removed as shown in *figure 1*.

The radius of the circle PQ is 6cm and PS = 2cm.
Angle QPR = 100°.

Calculate the area of the biscuit.

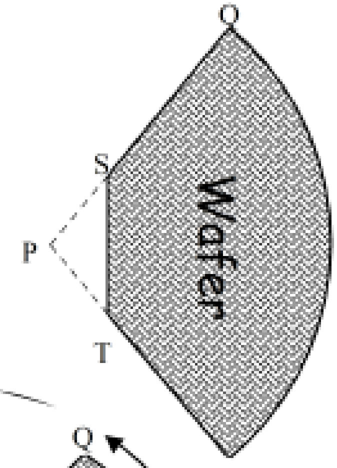


figure 1

(b) A smaller version of the wafer is produced.

In this smaller biscuit PQ = $4 \cdot 4$ cm .

Given that the arc length QR = $6 \cdot 9$ cm , calculate the size of angle QPR, correct to the nearest degree, for this version of the biscuit.

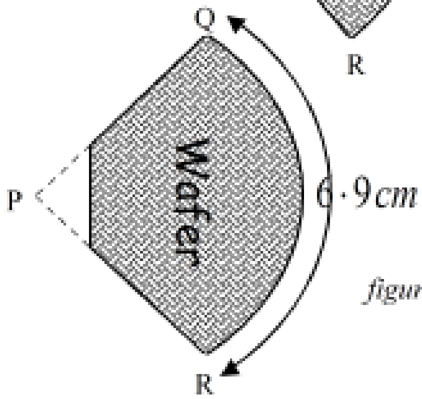


figure 2

2. A grandfather clock has a pendulum which travels along an arc of a circle, centre O.

The arm length of the pendulum is 60cm.

The pendulum swings from position OA to OB.
The length of the arc AB is 21cm.

Calculate the size of angle AOB to the nearest degree.

3. (a) A circle, centre O, has an arc PQ of length 40cm.
If the diameter of the circle is 80cm, calculate the size of angle POQ correct to 1 d.p.

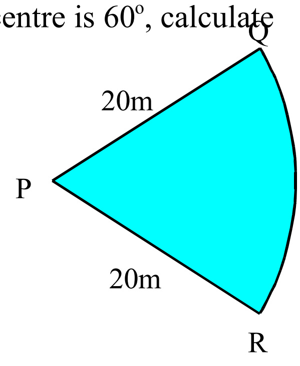
(b) A circle, centre O, has a sector EOF with an area of 50cm^2.
If the radius of the circle is 8cm, calculate the size of angle EOF correct to 1 d.p.

(c) An arc AB on a circle, centre O, has a length of 16mm.
If angle AOB = 75° , calculate the radius of this circle.

(d) A sector of a circle has an area of 12cm^2. If the angle at the centre is 60°, calculate the diameter of the circle correct to 2-decimal places.

4. The shape opposite is the sector of a circle, centre P, radius 20m.

The area of the sector is $251 \cdot 2$ square metres.



Find the length of the arc QR.

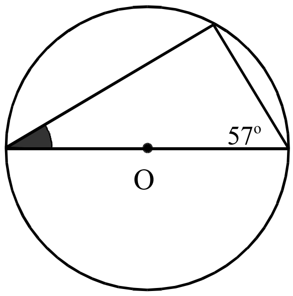
The Circle (4) - Angles in Circles

Copy and learn the following ....

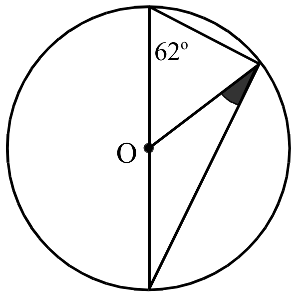
1. *Angles in a semi-circle equal 90º.*
2. *The angle between a tangent and a radius is 90º.*
3. *Look for isosceles triangles in all diagrams.*
4. *The angles in any triangle add up to 180º.*
5. *Two angles in a straight line add up to 180º.*
6. *Be aware of the tangent kite.*

Exercise1 : Find the size of the shaded angles in each diagram.
(O is the centre of each circle, PS and PT are tangents)

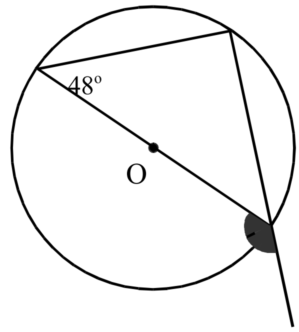
1.



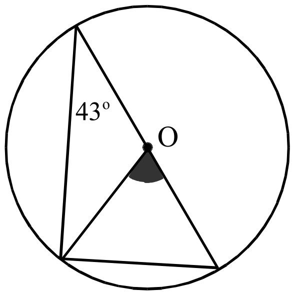
2.



3.



4.



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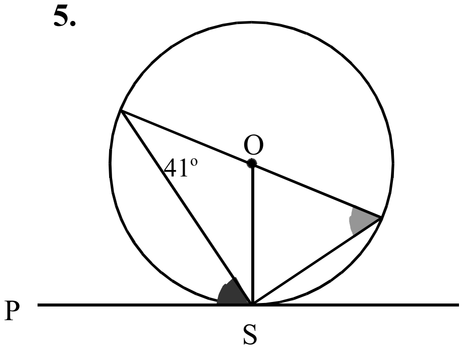

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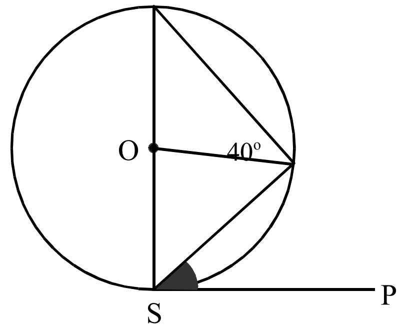

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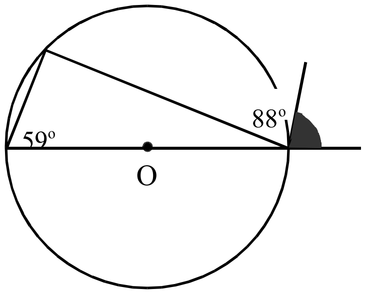

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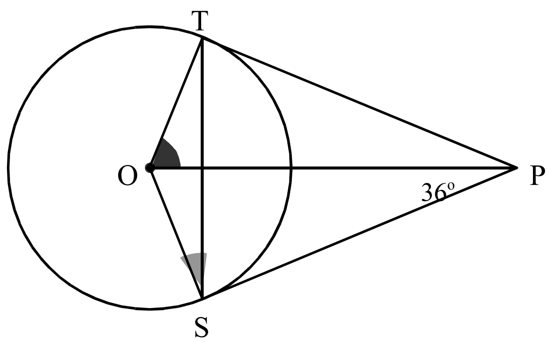
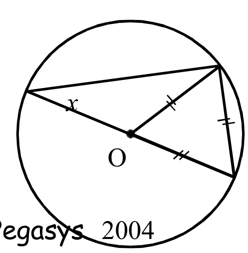
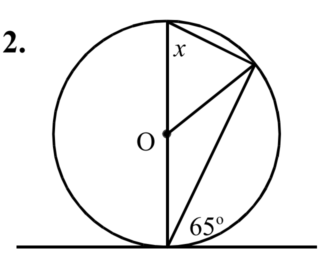


Exercise 2 : Calculate the size of angle *x* in each diagram below.

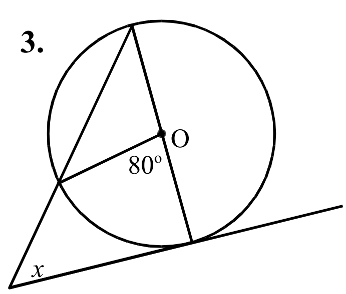
1.



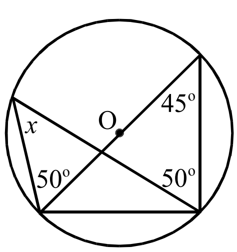
2.



3.



4.



Inequations

1. Solve each of the following inequations where x can only take values from the set of numbers ….. $\{-2, -1, 0, 1, 2, 3, 4, 5\}$.

(a) $6x + 2 \leq 3x + 5$ (b) $7x - 1 > 3x + 3$

(c) $3(2x + 1) \geq 5x + 8$ (d) $2(6 + 5x) < 8x + 12$

(e) $14 - 2(3 - x) \leq 8$ (f) $5 + 3(2 - x) \geq 14 - 6x$

(g) $2x - (4 - x) < x + 2$ (h) $3 - 4(2 + x) > 6(2 - x) - 17$

Example ………

answer … { 3 , 4 , 5 }

2. Solve each of the following inequations.

(a) $3a + 2 \leq 17 - 2a$ (b) $7(2x + 3) > 8x + 27$

(c) $2(5p - 12) \geq 7p - 18$ (d) $40 + 3k < 28 - k$

(e) $1 - 5(2 - m) \leq 2(m + 7)$ (f) $3(2y - 4) - 1 > 4(4 - y)$

(g) $2(3 - 4h) < 12 - 15h$ (h) $2 - 3(2 - x) > 2(1 - x) - 5$

Example ………

3. Solve each of the following inequations.

(a) $2a + 18 \leq 12 + 4a$ (b) $14 - 3x > x + 6$

(c) $3(p - 2) \geq 5p - 10$ (d) $16 - 3k < 20 - k$

(e) $7(2 - d) \leq 2(d - 12)$ (f) $2(2y - 1) - 8 > 10(1 + y)$

(g) $4(3 - 4h) < 12 + h$ (h) $3(2 - y) > 2(1 + 3y) - 7$

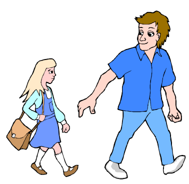
(i) $w(w - 3) > 5(w + 2) + w^2$ (j) $4d(1 + d) < 2d(2d + 3) - 20$

Example ………

4. I think of a whole number, treble it and subtract 3. The answer must be less than or equal to 12. Form an inequation and solve it to find the possible starting whole numbers.

5. I subtract a whole number from 8 and double the answer. The result must be greater than 10. Form an inequation and solve it to find the possible starting whole numbers.

6. Fred and Jane are brother and sister. Fred is 3 years older than twice Jane's age.



The sum of their ages is less than 36 years.

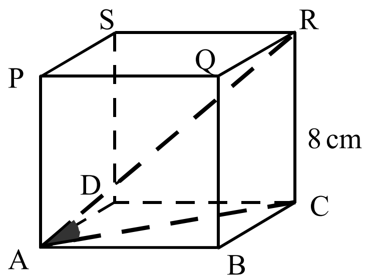
Taking Jane's age to be x years form an inequation. What can you say about Jane's age?

Trigonometry (1) - 3D Calculations

1. The diagram opposite is a **cube** of side 8cm.

   (a) Calculate the length of AC.

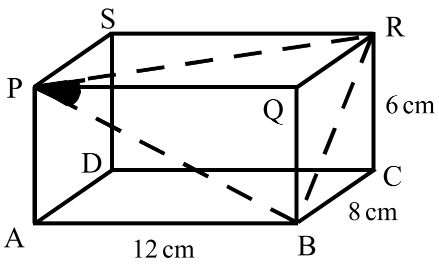
   (b) Hence calculate the size of angle CAR.



2. The cuboid opposite has dimensions 12cm by 8cm by 6cm as shown.

   (a) Calculate the lengths of PB and BR.

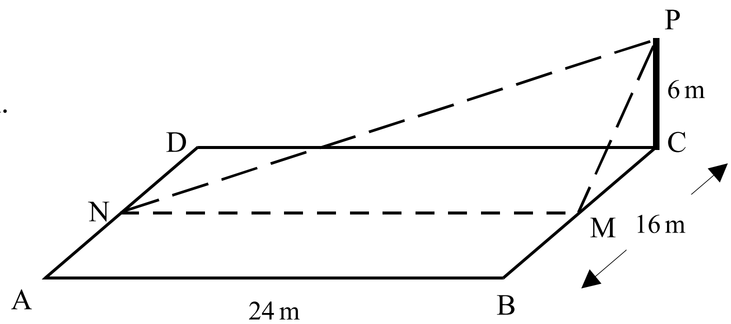
   (b) Hence calculate the size of $\angle BPR$



3. ABCD is a rectangle resting on a horizontal plane with AB = 24 m and BC = 16 m as shown.
   CP is a vertical post of length 6 m.
   M and N are the mid-points of BC and AD respectively.

   (a) Calculate the size of angle CMP.

   (b) Calculate the length of PM and hence calculate the size of angle MNP.



4. A cone of base radius 12 cm has a vertical height of 20 cm as shown.

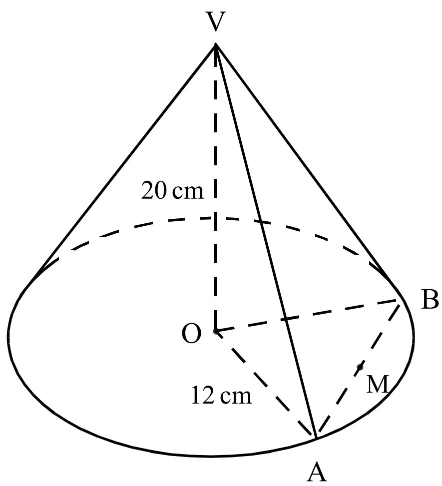
   (a) Calculate the slant height of the cone.

   (b) Calculate the size of angle OBV.

   (c) i) Given that M is the mid-point of AB and $\angle$ AOB = 90°, calculate the length of the chord AB.

   ii) Hence calculate the size of $\angle$ MBV

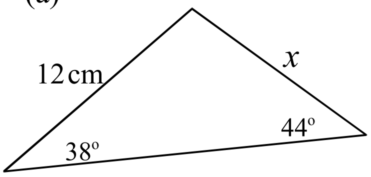
   iii) Calculate the area of triangle ABV.



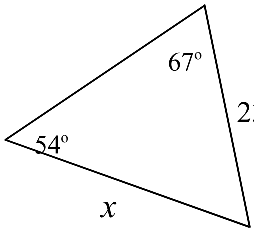
Trigonometry (2) - The Sine Rule

1. Use the *sine rule* to calculate the side marked x in each triangle below.

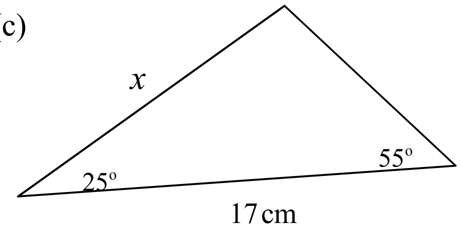
(a)



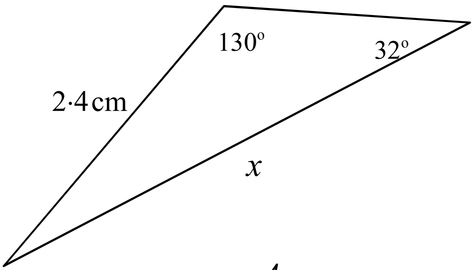
(b)



(c)



(d)



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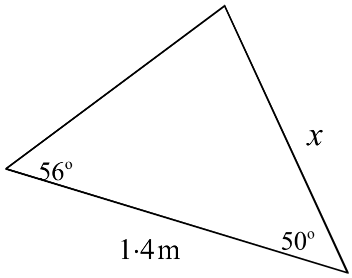
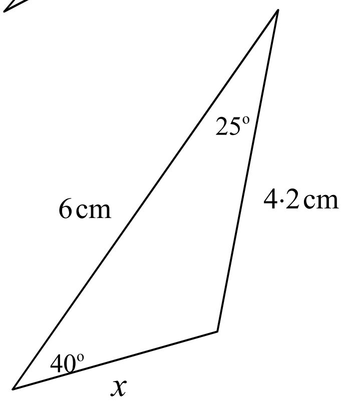
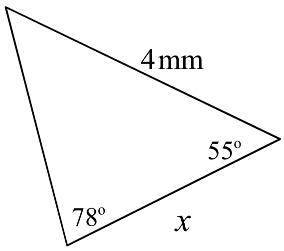


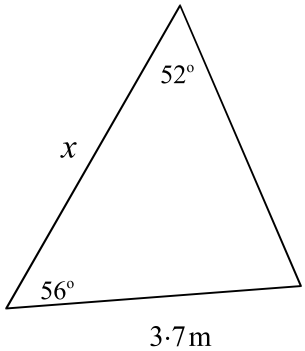
(f)



(g)

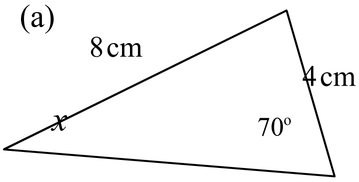


(h)

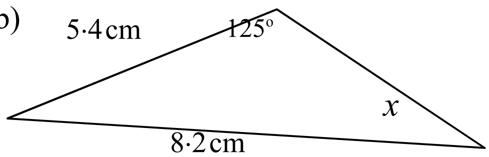


2. Use the *sine rule* to calculate the size of the angle marked x in each triangle below.

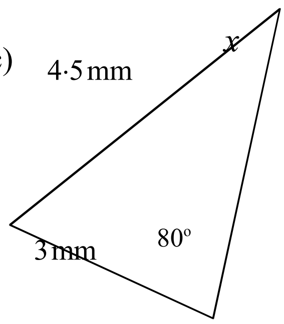
(a)



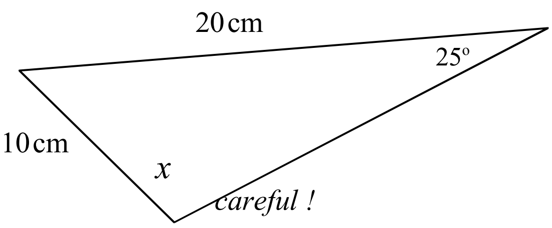
(b)



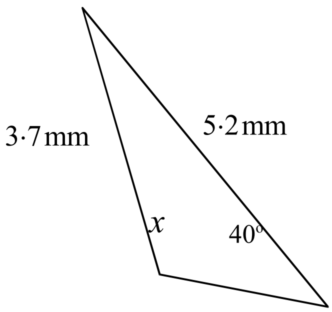
(c)



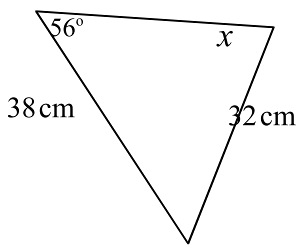
(d)



(e)



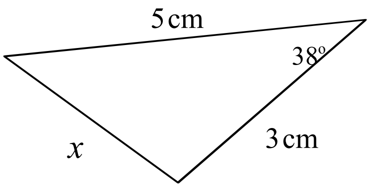
(f)



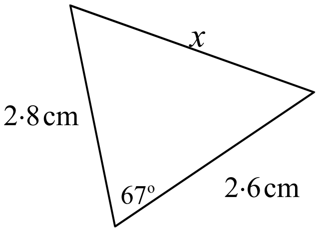
Trigonometry (3) - The Cosine Rule

1. Use the *cosine rule* to calculate the side marked x in each triangle below.

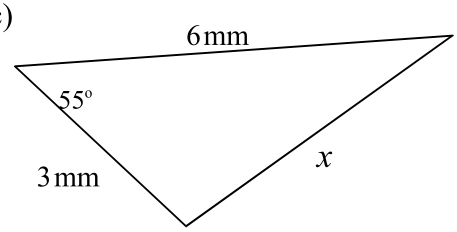
(a)



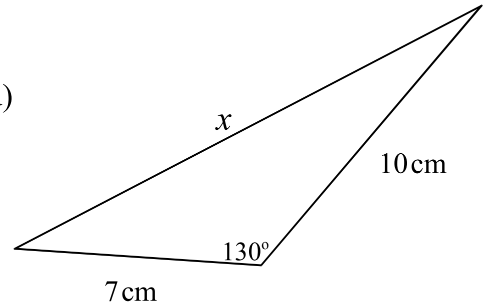
(b)



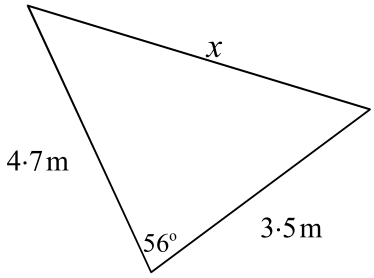
(c)



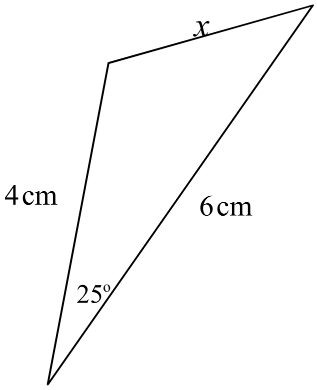
(d)



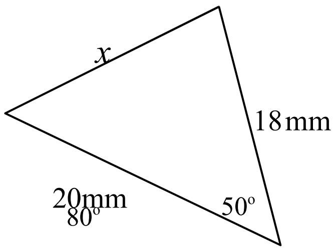
(e)



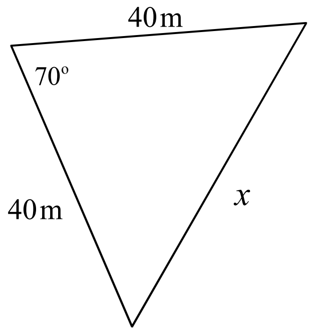
(f)



(g)

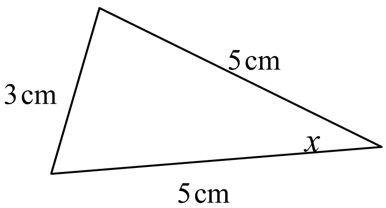


(h)

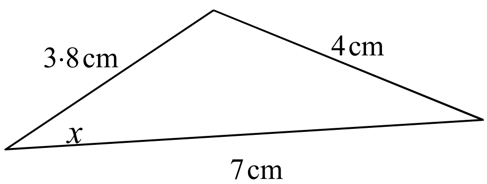


2. Use the 2nd form of the *cosine rule* to calculate the size of the angle marked x in each triangle below.

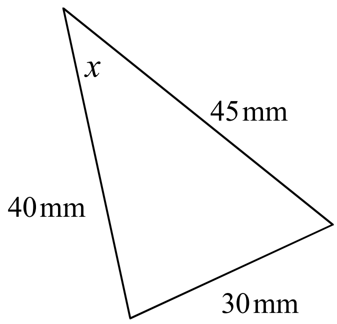
(a)



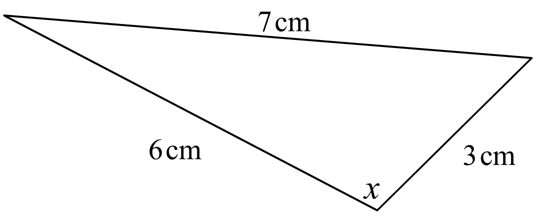
(b)



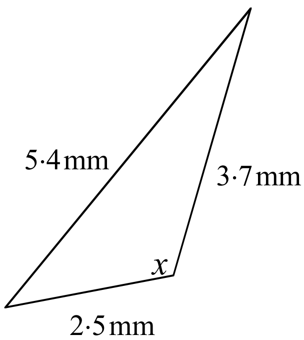
(c)



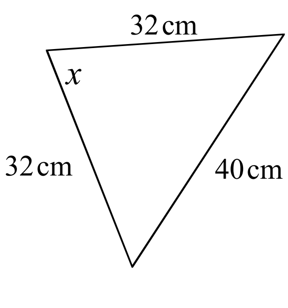
(d)



(e)



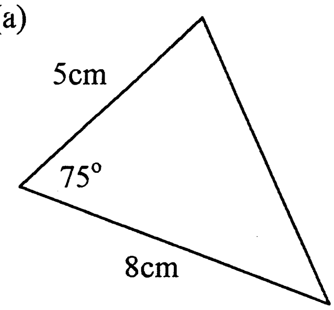
(f)



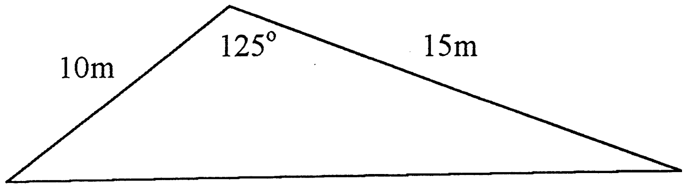
Trigonometry (4) - The Area of a Triangle

1. Use trigonometry to calculate the area of each triangle below.

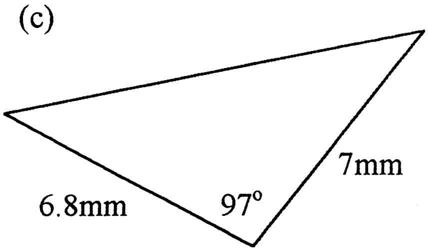
(a)



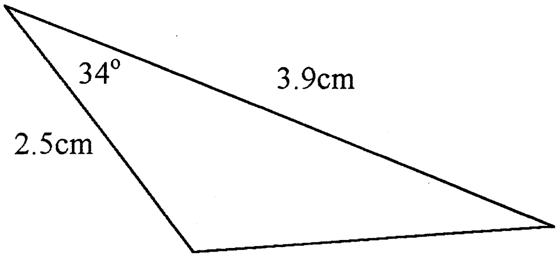
(b)



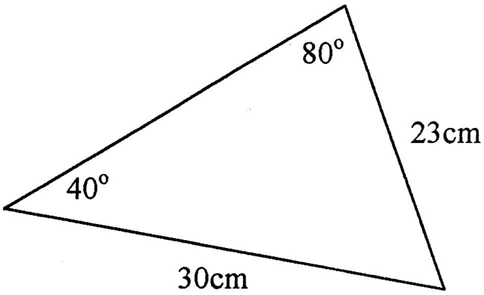
(c)



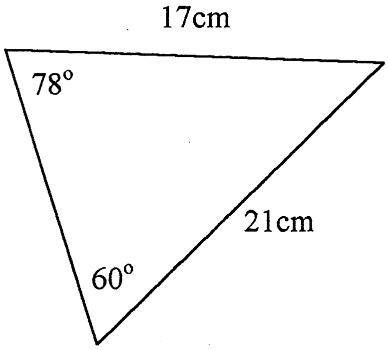
(d)



(e)

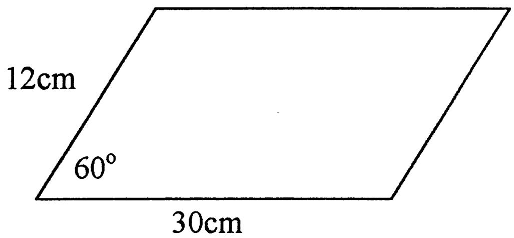


(f)

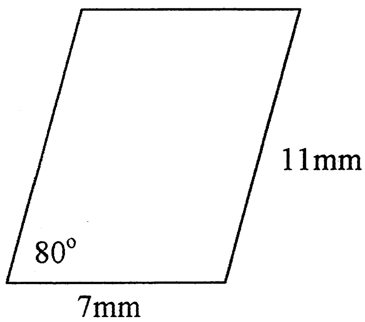


2. Calculate the area of each parallelogram below.

(a)

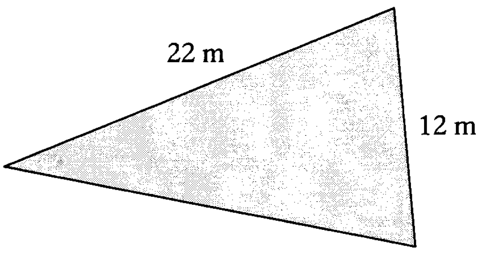


(b)



3. A fishing boat sets its net in the form of a triangle as shown opposite.
The total length of its net is 54 metres.

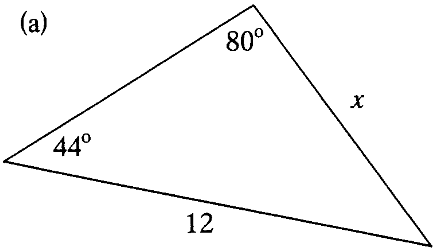
Calculate the surface area of the water enclosed within the net.



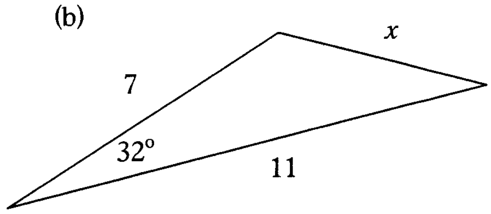
<u>Trigonometry</u> (5) - Mixed Exercise & Frameworks

1. Calculate the value of x in each triangle below.

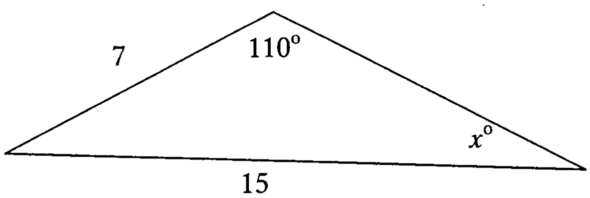
(a)



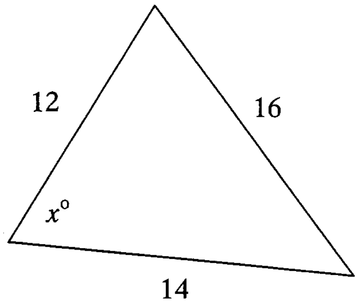
(b)



(c)



(d)

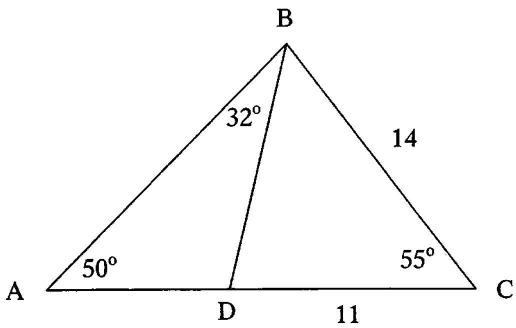


2. Calculate the area of the triangle with sides measuring 12cm, 14cm and 20cm.

3. (a) Calculate the length of BD.

   (b) Calculate the length of AD.

   (c) Calculate the area of triangle ABC



4. From the framework opposite:

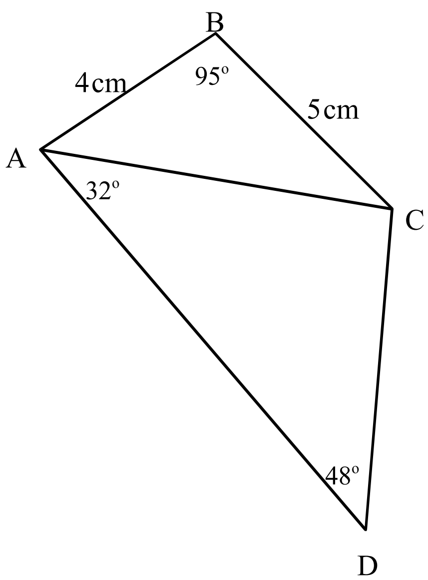
   (a) Calculate the length of AC.

   (b) Calculate the size of $\angle$BAC.

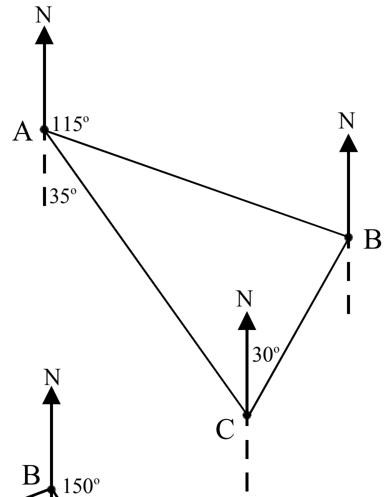
   (c) Write down the size of $\angle$ACD.

   (d) Calculate the length of AD.

   (e) Calculate the area of the quadrilateral ABCD.



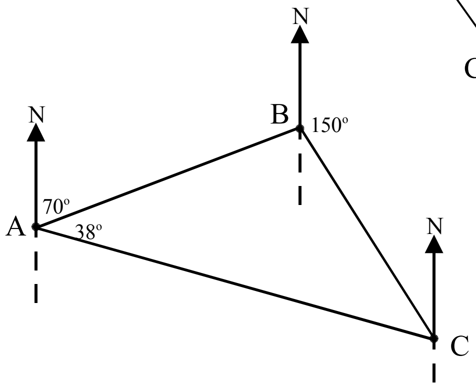
Trigonometry (6) - Bearings



1. (a) Copy the bearing diagram opposite and fill in as many angles as you can.

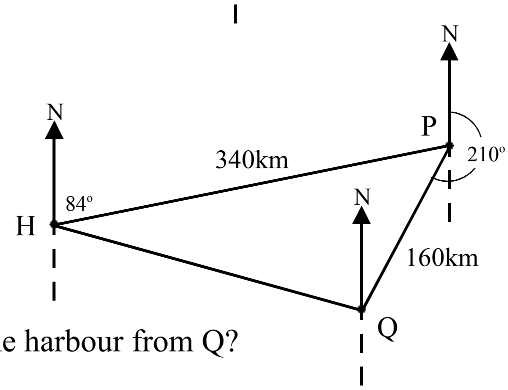
   (b) Now answer the following questions …..

   What is the bearing of … i) B from A
   ii) A from B
   iii) C from B
   iv) A from C
   v) C from A
   vi) B from C



2. Repeat question 1. for this bearing diagram.

3. A ship sails from harbour H on a bearing of 084° for 340km until it reaches point P. It then sails on a bearing of 210° for 160km until it reaches point Q.

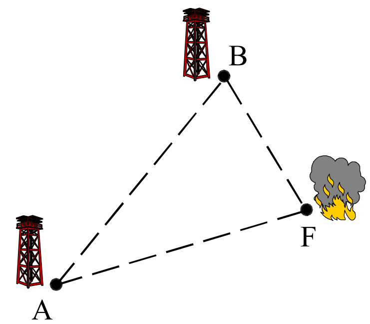


   (a) Calculate the distance between point Q and the harbour.

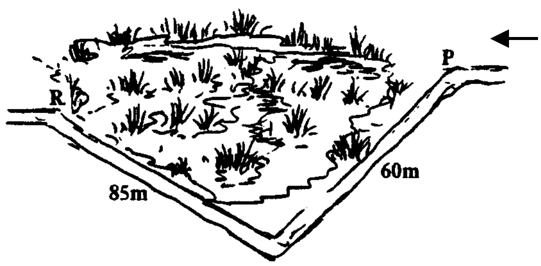
   (b) On what bearing must the ship sail to return directly to the harbour from Q?

4. A and B represent two forest look-out towers.
   A is 5km and on a bearing of 220° from B.
   A forest fire is sighted at F, on a bearing of 070° from A and 150° from B.

   A fire-fighting helicopter leaves A for F. What distance does this helicopter have to travel to reach the fire?



5. A surveyor is walking due west when he comes to a marsh. To avoid the marsh he turns at P and walks for 60 metres on a bearing of 215° and then for 85 metres on a bearing of 290°.
   He then calculates the distance PR, the direct distance across the marsh. What answer should he get?

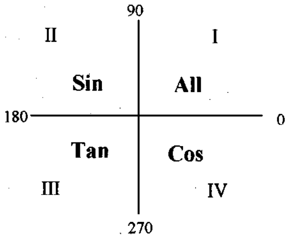


6. Two ships leave Liverpool at the same time. One of them travels north-west at an average speed of 10·5 km/h while the other travels at an average speed of 14 km/h on a bearing of 280°.

How far apart are these ships after 2 hours?

Trigonometry (7) - Trig Equations

Reminders



To Find Related Angles

Quadrant	I	II	II	IV
	x°	180 – x°	180 + x°	360 – x°
e.g.	30°	150°	210°	330°

example .... Solve the equation ......

$6\sin x^\circ - 1 = 0$, where $0 < x \le 360$

$6\sin x^\circ = 1$

$\sin x^\circ = \frac{1}{6}$ ($\sin$ *is* $+ve$ $\therefore$ *quad.* 1 *and* 2)

$\therefore\ x = 9\cdot6^\circ$ *or* $180 - 9\cdot6 = 170\cdot4^\circ$

1. Solve the following equations for $0 \le x < 360°$, giving answers correct to 1 decimal place.

(a) $3 \cos x° = 1$ (b) $2 \sin x° = -1$ (c) $4 \tan x° = 7$

(d) $5 \cos x° + 1 = 0$ (e) $4 \sin x° - 1 = 0$ (f) $5 \tan x° + 2 = 0$

(g) $7 \sin x° - 1 = 4$ (h) $\sqrt{2} \cos x° - 1 = 0$ (i) $2 \sin x° + \sqrt{3} = 0$

(j) $10 \tan x° - 5 = 14$ (k) $\sin(x + 30)° = 0.5$ (l) $\cos(x - 40)° = 0.6$

(m) $3 \tan (x - 15)° = 4$ (n) $6 \sin(x + 50)° = 5$

2. Solve the following equations for $0 \le x < 360°$, giving answers correct to 1 decimal place.

(a) $7 + 10 \cos x° = 12$ (b) $5 \sin x° + 3 = 5$

(c) $17 - 5 \cos x° = 20$ (d) $2 \tan x° + 3 = 5$

(e) $21 + 2 \cos x° = 20$ (f) $2 \sin x° - 1.6 = 0$

(g) $3 \cos x° + \sqrt{2} = 0$ (h) $4 \sin (x + 20)° = 1$

(I) $5 \cos (x - 15)° + 1 = 0$ (j) $2 \sin(x + 35)° = 1.2$

3. Solve each of the following for $0 \le x < 360$.

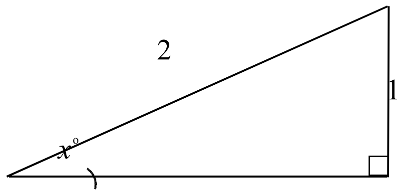
(a) $\sin^2 x^\circ = \frac{1}{9}$ (b) $3\cos^2 x^\circ = 1$

(c) $\tan^2 x^\circ - 4 = 0$ (d) $7\sin^2 x^\circ - 3 = 0$

Trigonometry (8) - Proofs & Identities

1. Consider the diagram opposite :

   (a) Write down the **exact** values of ..... $\sin x^\circ$, $\cos x^\circ$ and $\tan x^\circ$.

   (b) Prove that $\sin^2 x^\circ + \cos^2 x^\circ = 1$.

   (c) Show that $\dfrac{\sin x^\circ}{\cos x^\circ} = \tan x^\circ$.

2. (a) If $\cos a^\circ = \dfrac{1}{\sqrt{5}}$, find the **exact** values of $\sin a^\circ$ and $\tan a^\circ$ where $0 < a < 90$.

   (b) Prove that $\cos^2 a^\circ = 1 - \sin^2 a^\circ$.

   (c) Show that $\dfrac{\sin^2 a^\circ}{\cos^2 a^\circ} = \tan^2 a^\circ$.

   (d) Show that $2(3\sin a^\circ + 4\cos a^\circ) = 4\sqrt{5}$.

3. Prove each of the following trigonometric identities :

   (a) $3\cos^2 a + 3\sin^2 a = 3$

   (b) $(\cos x + \sin x)^2 = 1 + 2\sin x\cos x$

   (c) $(\cos x + \sin x)(\cos x - \sin x) = 2\cos^2 x - 1$

   (d) $\dfrac{\sin x}{\cos x} + \dfrac{\cos x}{\sin x} = \dfrac{1}{\cos x \sin x}$

   (e) $\tan^2 p - \tan^2 p \sin^2 p = \sin^2 p$

   (f) $\cos^4 x - \sin^4 x = 2\cos^2 x - 1$

   (g) $3\sin^2\theta + 2\cos^2\theta = 2 + \sin^2\theta$

   (h) $\tan\alpha + \dfrac{1}{\tan\alpha} = \dfrac{1}{\sin\alpha\cos\alpha}$

   (i) $\dfrac{\cos\beta}{1+\sin\beta} + \dfrac{1+\sin\beta}{\cos\beta} = \dfrac{2}{\cos\beta}$

4. Show that $(2\cos x + 5\sin x)^2 + (5\cos x - 2\sin x)^2 = 29$

5. Given that $p = \cos\theta + \sin\theta$ and $q = \cos\theta - \sin\theta$, prove that :

(a) $pq = 2\cos^2\theta - 1$ (b) $\sin^2\theta = \frac{1}{2}(1 - pq)$

Solving Quadratic Equations (1) - By Factorising

1. **Examples …….. Common Factor**

Solve

(a) $x^2 - 5x = 0$

$x(x-5) = 0$

$\therefore\ x = 0 \ \ or \ \ x - 5 = 0$

$x = 0 \ \ or \ \ x = 5$

(b) $4x^2 + 18x = 0$

$2x(2x+9) = 0$

$\therefore\ 2x = 0 \ \ or \ \ 2x + 9 = 0$

$x = 0 \ \ or \ \ x = -\frac{9}{2}$

Exercise 1 Solve the following quadratic equations.

(a)	$x^2 + 6x = 0$	(b)	$y^2 - 9y = 0$
(c)	$z^2 - z = 0$	(d)	$a^2 + 7a = 0$
(e)	$b^2 + 11b = 0$	(f)	$3x^2 - 9x = 0$
(g)	$4y^2 - 28y = 0$	(h)	$5z^2 - 60z = 0$
(i)	$7m^2 + 35m = 0$	(j)	$9n^2 - 54n = 0$
(k)	$6p^2 - 15p = 0$	(l)	$4q^2 - 14q = 0$
(m)	$15r^2 + 25r = 0$	(n)	$12u^2 + 16u = 0$
(o)	$18v^2 + 21v = 0$	(p)	$28l^2 - 35l = 0$
(q)	$16m^2 - 18m = 0$	(r)	$9n^2 + 16n = 0$
(s)	$8b^2 - 10b = 0$	(t)	$10c^2 - 45c = 0$

2. **Examples .. Difference of Two Squares**

Solve

(a) $y^2 - 9 = 0$

$(y+3)(y-3) = 0$

$\therefore\ y + 3 = 0 \ \ or \ \ y - 3 = 0$

$y = -3 \ \ or \ \ y = 3$

(b) $25a^2 - 49 = 0$

$(5a+7)(5a-7) = 0$

$\therefore\ 5a + 7 = 0 \ \ or \ \ 5a - 7 = 0$

$a = -\frac{7}{5} \ \ or \ \ a = \frac{7}{5}$

Exercise 2 Solve the following quadratic equations.

(a)	$x^2 - 36 = 0$	(b)	$y^2 - 100 = 0$
(c)	$z^2 - 49 = 0$	(d)	$a^2 - 1 = 0$
(e)	$b^2 - 64 = 0$	(f)	$x^2 - 121 = 0$
(g)	$4y^2 - 25 = 0$	(h)	$9z^2 - 16 = 0$
(i)	$25m^2 - 64 = 0$	(j)	$9n^2 - 25 = 0$
(k)	$25p^2 - 1 = 0$	(l)	$4q^2 - 9 = 0$
(m)	$49r^2 - 144 = 0$	(n)	$36u^2 - 25 = 0$
(o)	$81v^2 - 1 = 0$	(p)	$4l^2 - 1 = 0$
(q)	$16m^2 - 9 = 0$	(r)	$1 - 16p^2 = 0$
(s)	$25 - 4g^2 = 0$	(t)	$4 - (p+1)^2 = 0$

3. **Examples .. Tri-nomials**

Solve

(a) $x^2 - x - 6 = 0$

$(x-3)(x+2) = 0$

$\therefore\ x - 3 = 0 \ \ or \ \ x + 2 = 0$

$x = 3 \ \ or \ \ x = -2$

(b) $2a^2 - 7a + 6 = 0$

$(2a-3)(a-2) = 0$

$\therefore\ 2a - 3 = 0 \ \ or \ \ a - 2 = 0$

$a = \frac{3}{2} \ \ or \ \ a = 2$

Exercise 3 Solve the following quadratic equations.

(a)	$x^2 - 9x + 20 = 0$	(b)	$y^2 - 12y + 27 = 0$
(c)	$z^2 - 6z + 5 = 0$	(d)	$a^2 - 8a + 15 = 0$
(e)	$b^2 + 14b + 40 = 0$	(f)	$x^2 + 10x + 9 = 0$
(g)	$y^2 + 18y + 81 = 0$	(h)	$z^2 + 2z - 48 = 0$
(i)	$m^2 + 7m - 60 = 0$	(j)	$n^2 + 5n - 36 = 0$
(k)	$p^2 + p - 56 = 0$	(l)	$q^2 - 4q - 21 = 0$
(m)	$r^2 - 6r - 72 = 0$	(n)	$u^2 - 7u - 8 = 0$
(o)	$3v^2 + 10v + 7 = 0$	(p)	$2l^2 - 11l + 5 = 0$
(q)	$12m^2 - 31m + 7 = 0$	(r)	$3n^2 - 19n + 28 = 0$
(s)	$4b^2 - 20b + 25 = 0$	(t)	$9c^2 + 18c + 8 = 0$
(u)	$3q^2 + 14q - 5 = 0$	(v)	$6a^2 + a - 12 = 0$
(w)	$8b^2 - 2b - 15 = 0$	(x)	$2c^2 - 5c - 18 = 0$
(y)	$12m^2 - 8m - 15 = 0$	(z)	$2n^2 - n - 28 = 0$

Solving Quadratic Equations (2)

1. Solve each of the following quadratic equations by first removing a common factor and then factorising further.

(a)	$2x^2+8x-10=0$	(b)	$4y^2+20y+24=0$
(c)	$5z^2-5z-60=0$	(d)	$2a^2-6a-56=0$
(e)	$7b^2-70b+63=0$	(f)	$10x^2-20x-150=0$
(g)	$2y^2-18=0$	(h)	$6z^2-6=0$
(i)	$6m^2+21m+9=0$	(j)	$12n^2+20n+8=0$
(k)	$5p^2-45=0$	(l)	$48q^2+8q-16=0$
(m)	$10r^2-60r-720=0$	(n)	$28u^2-63=0$
(o)	$36v^2+120v+84=0$	(p)	$80l^2+40l-400=0$
(q)	$27m^2-12=0$	(r)	$6n^2-38n+56=0$
(s)	$18b^2+24b+8=0$	(t)	$60c^2+75c-90=0$
(u)	$24q^2-22q+4=0$	(v)	$54a^2+9a-108=0$
(w)	$18b^2-39b+18=0$	(x)	$250c^2-10=0$
(y)	$60m^2-21m-18=0$	(z)	$36n^2-30n-24=0$

Examples .. Mixed Factors
Solve

(a)
$$3x^2-6x-24=0$$
$$3(x^2-2x-8)=0$$
$$3(x-4)(x+2)=0$$
$$\therefore\ x-4=0 \ \ or \ \ x+2=0$$
$$x=4 \ \ or \ \ x=-2$$

(b)
$$80a^2-20=0$$
$$20(4a^2-1)=0$$
$$20(2a-1)(2a+1)=0$$
$$\therefore\ 2a-1=0 \ \ or \ \ 2a+1=0$$
$$a=\tfrac{1}{2} \ \ or \ \ a=-\tfrac{1}{2}$$

2. Use the quadratic formula, $x=\dfrac{-b\pm\sqrt{(b^2-4ac)}}{2a}$, to solve the following quadratic equations rounding your answers to 1-decimal place.

[**useful tip** ……. *calculate* $b^2-(4ac)$, *separately, away from the equation, and bring the final number back in ….. why have we introduced the extra brackets*?]

(a)	$x^2+4x+1=0$	(b)	$x^2+6x+4=0$	(c)	$x^2+7x+5=0$
(d)	$x^2+3x-1=0$	(e)	$x^2-6x+3=0$	(f)	$x^2-4x-7=0$
(g)	$2x^2+x-4=0$	(h)	$2x^2-3x-4=0$	(i)	$2x^2+12x+9=0$
(j)	$4x^2-12x+3=0$	(k)	$3x^2+x-5=0$	(l)	$7x^2+3x-1=0$

3. Solve each of the following quadratic equations by "completing the square".
Round your answers to 2-decimal places.

(a)	$x^2+4x+2=0$	(b)	$x^2+6x-2=0$	(c)	$x^2+2x-1=0$
(d)	$x^2+8x+3=0$	(e)	$x^2-6x-4=0$	(f)	$x^2-12x+21=0$
(g)	$2x^2+4x-3=0$	(h)	$2x^2-8x+5=0$	(i)	$x^2+5x-2=0$?

4. ***Extension*** …… Solve the following equations using the most appropriate method.

(a)	$(x+1)(x+2)=12$	(b)	$x(x-1)=3(x+1)$	(c)	$x(x+1)+3(x-2)=4(2x-1)$
(d)	$2(a^2-3)=a(a+1)$	(e)	$(x+3)\ +\ \dfrac{6}{(x-4)}\ =\ 0$	(f)	$(2x-1)^2-3=6$

Quadratic Graphs

1. Sketch the graphs of the following quadratic functions marking all relevant points.

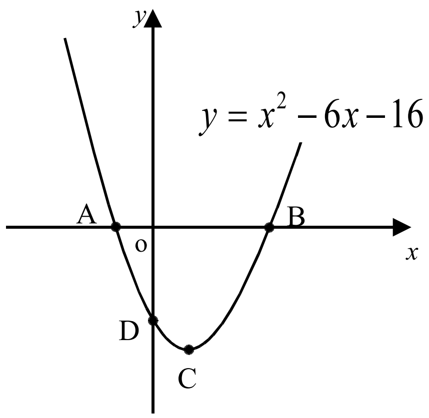
   Then …. for each function answer the following questions …

   i) State the *roots* (<u>or</u> zeros) of the function;
   ii) write down the equation of the *axis af symmetry*;
   iii) state the *coordinates* and *nature* of the turning point;
   iv) give the coordinates of the *y-intercept* point;
   v) state the *range* of the function.

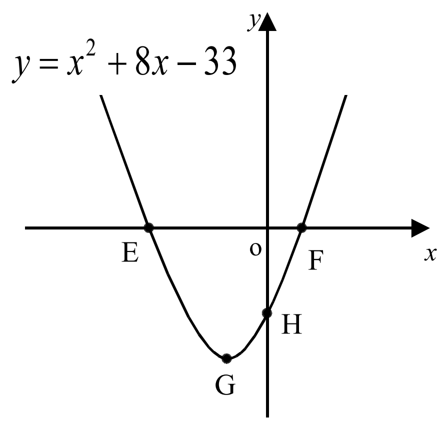
(a) $f(x) = x^2 + 2x - 3$	(b) $g(x) = x^2 - 2x - 8$	(c) $h(x) = x^2 - 4x - 5$
(d) $f(x) = x^2 + 6x$	(e) $g(x) = x^2 - 4x$	(f) $h(x) = 8x - x^2$
(g) $f(x) = 8 - 2x - x^2$	(h) $g(x) = 7 + 6x - x^2$	(i) $h(x) = x^2 - 10x + 21$
(j) $f(x) = x^2 - 3x - 4$	(k) $g(x) = x^2 + 7x + 6$	(l) $h(x) = 5x - x^2$
(m) $f(x) = 10 - 3x - x^2$	(n) $g(x) = 16 - x^2$	(o) $h(x) = x^2 - 9$

2. Find the coordinates of the points marked with letters in the diagrams below.

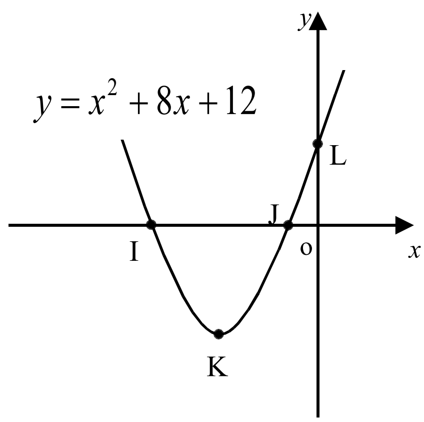
(a)



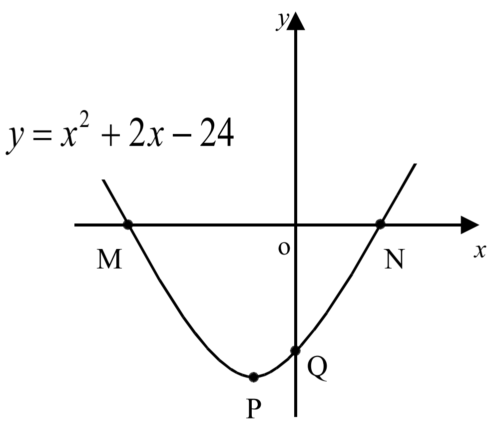
(b)



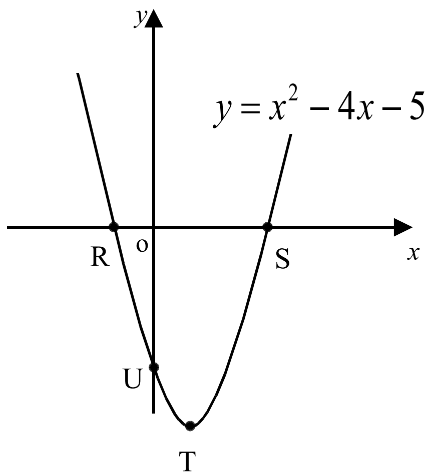
(c)



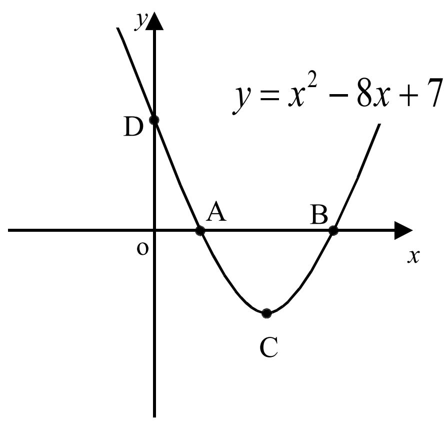
(d)



(e)



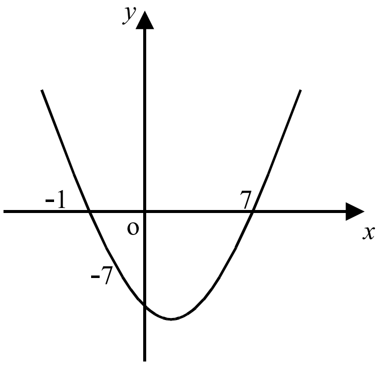
(f)



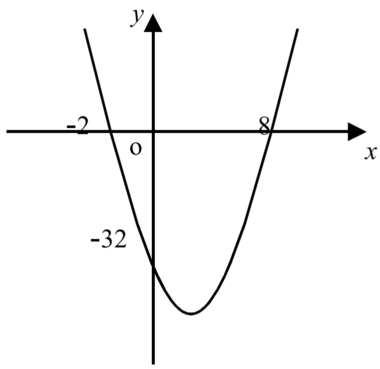
Establishing the Equation of a Quadratic Graph

1. Establish the equation connecting x and y for each graph below.

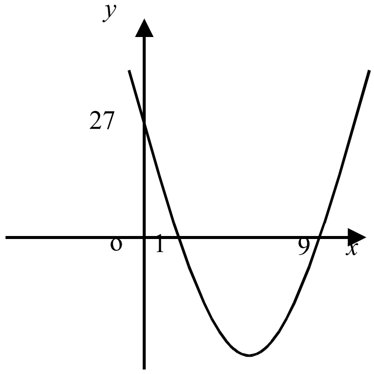
(a)



(b)

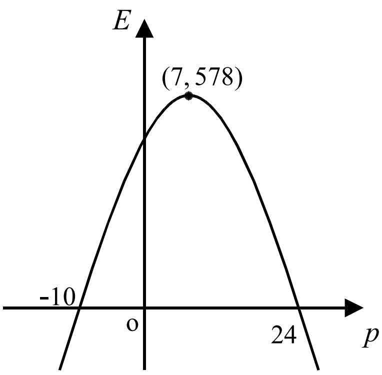


(c)

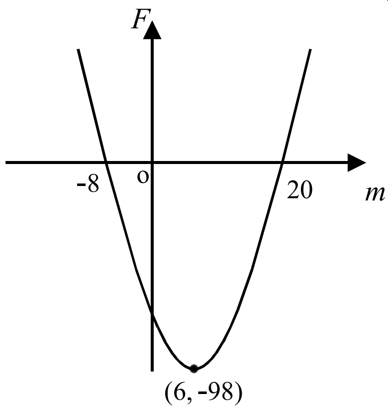


2. Establish the relationship between the two variables in each graph below.

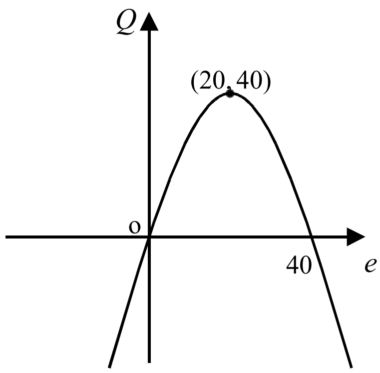
(a)



(b)



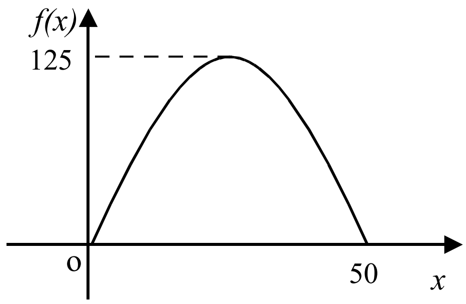
(c)



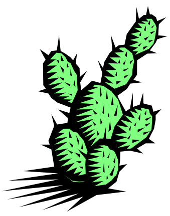
3. An arrow is fired into the air. Its trajectory is shown in the diagram opposite. Both the horozontal and vertical scales are in metres.

The arrow's flight-path is represented by the function f.

Establish the function f and hence calculate the height of the arrow above the ground when it is 15 metres horizontally from its firing point.



4.

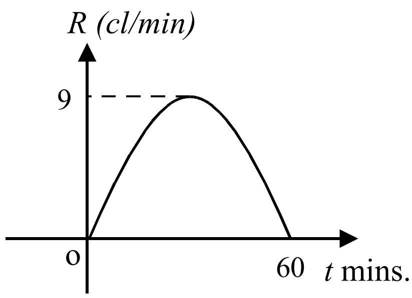


The rate, R, by which a cactus can absorb water, in centilitres/min, is shown in the graph.

Establish a formula for the rate R in terms of the time t minutes.

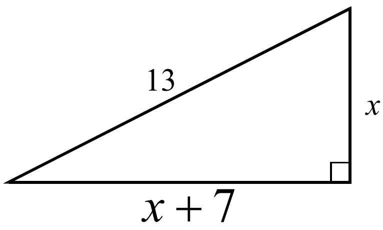
Hence calculate the absorbtion rate at :

(a) $t = 7$ (b) $t = 40$.



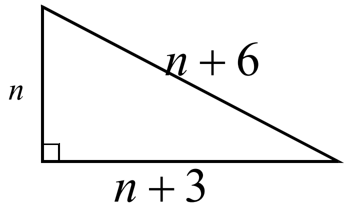
Problems Involving Quadratic Equations

1. The diagram opposite shows a right-angled triangle with sides measuring $13, x+7$ and x centimetres.



   (a) Form an equation and solve it to find x.

   (b) Hence calculate the perimeter of the triangle.

2. 

   The sides of a right angled triangle are n, $n+3$ and $n+6$ centimetres long.

   Find n by solving an equation, and hence calculate the area of this triangle.

3. Repeat question 2. for a right angled triangle with short sides measuring n and $2n+4$ and the hypotenuse measuring $3n-4$ millimetres.

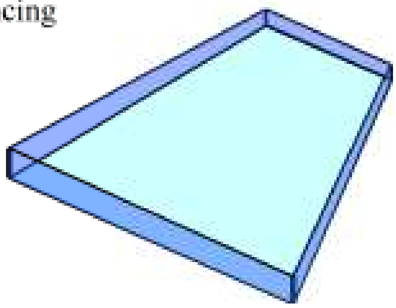
4. A rectangular sheet of glass has an area of $1500\,\text{cm}^2$ and a perimeter of $160\,\text{cm}$.

   (a) Taking the length of the glass sheet as x, write down an expression for the width of the sheet in terms of x.

   (b) Form an equation in x for the area of the sheet and solve it to find x.

   Hence state the dimensions of the glass sheet.

5. A farmer has 260 metres of clear plastic fencing. He uses all the fencing to create a rectangular holding pen.



   (a) Taking x as the length of the pen, write down an expression for the width of the pen in terms of x.

   (b) Given that the area of the pen is 4000 square metres, form an equation and solve it to find x, the length of the pen.

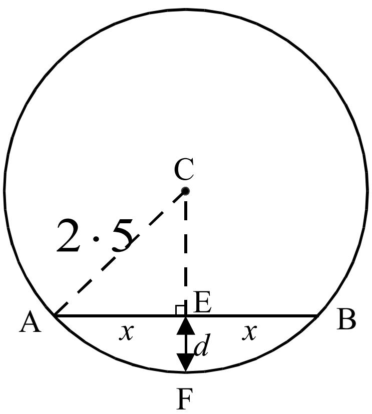
6. The perimeter of a rectangular lawn is 42 metres and its area is 80 square metres.

   Find the length and breadth of the lawn.

7. The diagram opposite shows the cross section of a cylindrical oil tank of radius 2·5 metres.

   The width AB of the exposed oil surface is $2x$ metres.

   The maximum depth of the oil is d metres, as shown.



   (a) Write down the length of CE in terms of d.

   (b) By applying Pythagoras' Theorem, show that …

$$x^2 - 5d + d^2 = 0$$

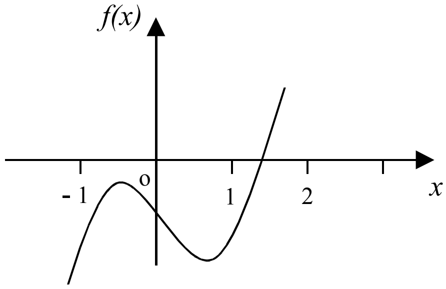
   (c) Find d when $x = \frac{3}{2}$, and $d < 1$.

Iteration (approximate roots)

1. Opposite is part of the graph of $f(x) = x^3 - x - 1$.

   The function has a root between $x = 1$ and $x = 2$.

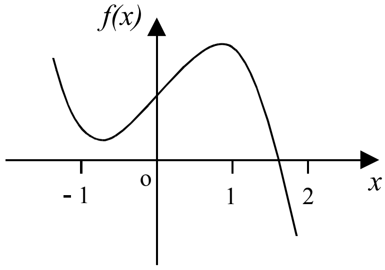
   Use iteration to find this root correct to 1-decimal place.
   You must show all your working clearly.



2. Opposite is part of the graph of $f(x) = 2x - x^3 + 2$.

   The function has a root between $x = 1$ and $x = 2$.

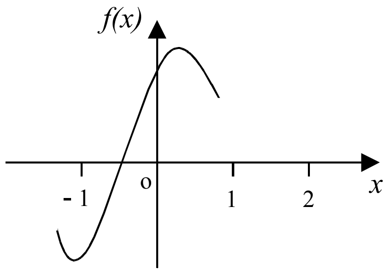
   Use iteration to find this root correct to 1-decimal place.
   You must show all your working clearly.



3. Opposite is part of the graph of $f(x) = 3x - 2x^3 - x^2 + 1$.

   The function has a root between $x = -1$ and $x = 0$.

   Use iteration to find this root correct to 1-decimal place.
   You must show all your working clearly.



4. Consider the equation $x^3 + 4x^2 - 10 = 0$.

   (a) Show that this equation has a root between $x = 1$ and $x = 2$.

   (b) Hence find this root correct to 1-decimal place.

5. Consider the cubic equation $x^3 - 4x^2 + 3 = 0$.

   (a) Show that this equation has a root between $x = 3$ and $x = 4$.

   (b) Hence find this root correct to 1-decimal place.

6. Consider the cubic equation $x^3 + x^2 - 7x + 6 = 0$.

   (a) Show that this equation has a root between $x = -4$ and $x = -3$.

   (b) Hence find this root correct to 1-decimal place.

7. Find a root near $x = 1$ for the cubic equation $x^3 - \frac{1}{2}x^2 - \frac{1}{2}x - 2 = 0$ giving your answer correct to 1-decimal place.

Standard Deviation

All of the following questions refer to chosen samples.

The recommended formulae for mean and standard deviation of a sample is shown opposite although alternative formulae may be used for this worksheet.

(mean) $\bar{x} = \dfrac{\sum x}{n}$, $s = \sqrt{\dfrac{\sum x^2 - \dfrac{(\sum x)^2}{n}}{n-1}}$

---

1. The heights, in centimetres, of a sample of ten tomato plants were recorded.

   The data gave the following summary totals:

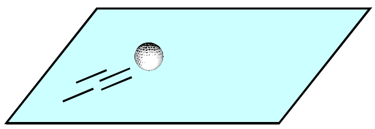
   $\sum x = 320$ $\sum x^2 = 11526$

   (a) Calculate the mean and standard deviation, giving your answer correct to 1-decimal place where necessary.

   (b) Another sample of ten plants has a mean height of 28cm and a standard deviation of 6·3. How does this second sample compare with the first?

2. A sample of twelve golf balls were artificially tested by rolling them across a flat area. The distance travelled, in metres, by each ball, was recorded.

   The data gave the following summary totals:

   $\sum x = 48$ $\sum x^2 = 192 \cdot 38$

   (a) Calculate the mean and standard deviation, giving your answer correct to 2-decimal places where necessary.

   (b) Another sample of twelve balls has a mean distance of 4·3m and a standard deviation of 0·81. How, with consistency in mind, does this second sample compare with the first?

3. A group of students took part in an end of year assessment. The marks out of fifty for a sample of these students are listed below.

   35 38 21 44 48 32 45 45 39 31 40

   (a) Calculate, correct to 3 significant figures, the mean and standard deviation of this sample.

   (b) A "highly commended" certificate is to be given to any student who has a mark that is more than one standard deviation above the mean.

   How many students in this sample would receive this certificate?

4. A sample of fifteen matchboxes was chosen from an assembly line. The number of matches in each box was recorded. The results are shown below.

   65 63 66 62 61 61 62 62
   63 64 59 61 60 63 64

   (a) Calculate, correct to 3 significant figures, the mean and standard deviation of this sample.

   (b) Boxes which contain **less than** one standard deviation of matches below the mean are to be discarded and refilled. How many boxes in this sample would be discarded?

Variation (1)

Direct Variation

1. Re-write each of the following statements by first using symbols and then as an equation involving k, the constant of variation.

   (a) C varies directly as t.   (b) V varies directly as the square of r.

   (c) The time (T) of swing of a simple pendulum varies as the square root of its length (l).

   (d) The volume (V) of a spherical object varies as the cube of its radius (r).

2. If $y \propto x$ and $y = 4$ when $x = 2$, find the constant of variation k.

   Hence find y when $x = 5$.

3. If y varies as the square of x and if $y = 3$ when $x = 2$, find the value of y when $x = 12$.

4. The value of a diamond (v) varies as the square of its weight (w).

   Given that a diamond weighing 2 carats is worth £1200, find the value of a diamond of similar quality which weighs 5 carats.

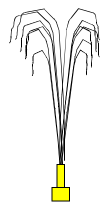
5. The time of vibration (T) of a pendulum varies as the square root of its length (l).

   Given that the length of a pendulum with a vibration time of 2 seconds is 100 cm, find the time of vibration of a pendulum measuring 64 cm.

6. The height (H) reached by a small fountain of water varies as the square of the water's velocity (v) leaving the nozzle.

   A velocity of 2 metres per second causes a height of 14 cm.

   What height is reached by a water velocity of 4 metres per second?



Inverse Variation

1. Re-write each of the following statements by first using symbols and then as an equation involving k, the constant of variation.

   (a) N varies inversely as p.   (b) V varies inversely as the square root of h.

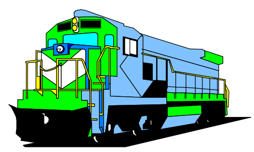
   (c) The resistence (e) of a wire varies inversely as the square of its diameter (d).

2. If y varies inversely as the cube of x and $y = 16$ when $x = 2$, find y when $x = 4$.

3. The maximum speed (v) at which a railway engine can travel, on the flat, varies inversely as the square of the number of loaded wagons (n) attached to it.

   With four wagons attached, the train has a maximum speed of 40 km/h.

   What is the maximum speed of the train when 8 wagons are attached?



4. The time (T) of one oscillation of a vibrating magnetometer varies inversely as the square root of the horizontal intensity (h) of the earth's magnetic field.

   In Iceland, $T = 0 \cdot 75$ seconds when $h = 0 \cdot 12$ gauss.

   Find the time of one oscillation in Hollywood where $h = 0 \cdot 25$ gauss.

Variation (2) - Joint Variation

1. Re-write each of the following variation statements by first using symbols and then as an equation involving k, the constant of variation.

   (a) E varies directly as t and inversely as p^2.
   (b) Q varies directly as the square of h and t^3.
   (c) M varies as e and g^2 and inversely as $\sqrt{d}$.
   (d) T varies as m, p and g and inversely as s^2.
   (e) The volume (V) of a cone varies jointly as the height (h) of the cone and the square of the radius (r) of the base.
   (f) The time of oscillation (T) of a pendulum varies directly as the square root of the length (l) and inversely as the square of the gravitational constant (g).

2. Given that $x \propto yz$ and $x = 24$ when $y = 3$ and $z = 2$, find the value of x when $y = 2$ and $z = 6$.

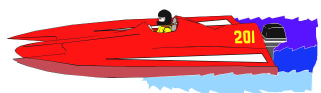
3. If x varies directly as y and inversely as z, and if $x = 8$ when $y = 4$ and $z = 3$, find the value of x when $y = 5$ and $z = 2$.

4. If a varies directly as b and inversely as the square root of c, and if $a = 9$ when $b = 3$ and $c = 4$, find the equation connecting a, b and c and hence find a when $b = 5$ and $c = 9$.

5. A number e varies directly as f and inversely as the square of g.

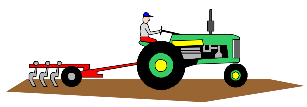
   (a) If $e = 10$ when $f = 5$ and $g = 4$, express e in terms of f and g.
   (b) Hence find the value of e when $f = 8$ and $g = 2$.

6. The air resistance (R) to an outboard engine running at certain speeds varies jointly as the area (a) of the surface exposed and as the square of the speed of the engine (s).
   When the engine is travelling at a speed of 50 m.p.h. with a surface area of 40 square centimetres exposed, the resistance to the engine is 3000 lb. wt.

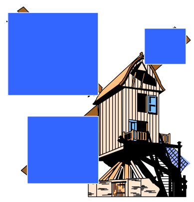
   

   Find the resistance to the engine if the surface exposed is 25 square centimetres and the speed is 60 m.p.h.

7. The time (T) required to plough a field varies directly as its area (A) and inversely as the number of men (n) employed.
   If 10 men plough a 15 hectare field in 3 days, how many men would be required to plough a field of 12 hectares in 4 days?

8. The horse power (P) of a windmill varies directly as the total sail area (A) and as the cube of the velocity of the wind (v).
   If the sail area is 150 square metres and the velocity of the wind is 20 km/h, the horse power is 35.

   Calculate the horse power generated by a windmill with a sail area of 200 square metres when the velocity of the wind is 30 km/h.

9. The volume (V) of a right circular cone varies jointly as its height (h) and as the square of its base radius (r). The volume of a cone which is 7 cm high with a base radius of 3 cm is 66 cm^3.

   (a) Show that the constant of variation (k) is equal to $\frac{22}{21}$.
   (b) Hence find the volume of a cone which is twice as high standing on a base with a radius half as large as the previous one.

Equations Involving Fractions (1)

1. Solve the following equations.

(a) $\frac{x}{8}=3$ (b) $\frac{5a}{2}=10$ (c) $\frac{3p}{4}=9$ (d) $\frac{w}{6}=\frac{5}{2}$

(e) $\frac{3x}{4}=\frac{9}{2}$ (f) $\frac{4x}{7}=\frac{3}{2}$ (g) $\frac{5h}{8}=\frac{1}{2}$ (h) $\frac{7p}{3}=\frac{4}{5}$

(i) $6=\frac{30}{x}$ (j) $14=\frac{42}{m}$ (k) $8=\frac{32}{3g}$ (l) $\frac{3}{2}=\frac{24}{x}$

(m) $\frac{6}{5}=\frac{12}{5k}$ (n) $\frac{9}{h}=\frac{3}{4}$ (o) $\frac{4}{3x}=\frac{2}{15}$ (p) $\frac{5}{4v}=\frac{2}{9}$

2 Solve each of the following equations.

(a) $\frac{x}{2}+3=11$ (b) $\frac{3p}{2}+1=4$ (c) $1+\frac{5d}{2}=16$

(d) $7+\frac{3q}{4}=13$ (e) $\frac{2y}{3}-8=2$ (f) $\frac{7e}{5}-4=10$

(g) $\frac{1}{2}a+6=14$ (h) $\frac{3}{4}x-7=11$ (i) $12+\frac{2}{3}y=16$

(j) $\frac{x}{4}+\frac{1}{3}=2$ (k) $\frac{p}{2}-\frac{3}{7}=1$ (l) $\frac{2w}{5}+3=\frac{7}{2}$

(m) $\frac{3y}{8}-1=\frac{2}{3}$ (n) $\frac{2h}{3}=1-\frac{h}{6}$ (o) $\frac{a}{4}=\frac{5}{2}-\frac{a}{3}$

(p) $\frac{2p}{3}-\frac{p}{4}=\frac{7}{12}$ (q) $\frac{4d}{9}-\frac{2}{3}=\frac{d}{2}$ (r) $\frac{2}{x}-\frac{1}{3}=\frac{1}{x}$

(s) $\frac{3}{4}-\frac{4}{y}=1$ (t) $\frac{1}{3e}+\frac{1}{e}=\frac{1}{3}$ (u) $\frac{1}{4k}+\frac{1}{3}=\frac{2}{5}$

3. Solve each of the following equations.

(a) $\frac{x+1}{4}+\frac{x+2}{5}=2$ (b) $\frac{2p+1}{3}+\frac{p+4}{5}=2$ (c) $\frac{x-1}{5}+2=\frac{2x-3}{3}$

(d) $\frac{2x+1}{4}+\frac{1}{2}=\frac{6x-1}{2}$ (e) $\frac{3y+4}{2}-y=\frac{2y+8}{3}$ (f) $\frac{d-1}{3}+\frac{3d-2}{4}=\frac{4}{3}$

(g) $\frac{3x+1}{5}+\frac{2x+5}{2}=-\frac{1}{2}$ (h) $3+\frac{3a+2}{3}=\frac{9a+5}{2}$ (i) $2w+\frac{w-1}{2}=\frac{5w+3}{3}$

Equations Involving Fractions (2)

1. Solve each of the following equations.

(*a*) $\frac{2}{5}x + 5 = 13$ (*b*) $\frac{1}{4} + \frac{4}{3}p = \frac{33}{4}$ (*c*) $\frac{2}{7}y - \frac{1}{2} = \frac{7}{2}$

(*d*) $2d + \frac{5}{8} = \frac{1}{2}$ (*e*) $\frac{8}{3} + 3h = \frac{5}{6}$ (*f*) $4c - \frac{11}{4} = \frac{7}{6}$

(*g*) $3(x+\frac{1}{2}) = \frac{5}{4}$ (*h*) $2(3p-\frac{5}{3}) + \frac{1}{6} = 1$ (*i*) $3k + \frac{4}{9} = \frac{1}{2}k + \frac{22}{3}$

2. Solve the following equations.

(*a*) $\frac{2}{3}(x+12) = 10$ (*b*) $\frac{3}{4}(3p+2) = 15$ (*c*) $\frac{7}{8}(5k-3) = 28$

(*d*) $\frac{4}{5}(2d+5)+3 = 23$ (*e*) $\frac{3}{5}(3p-1)-7 = 5$ (*f*) $\frac{5}{6}(h+5)-\frac{1}{2} = \frac{27}{2}$

(*g*) $\frac{1}{2}(x+1) + 2x = \frac{2}{3}(4x+1)$ (*h*) $\frac{1}{6}a + \frac{2}{3}(a+3) = \frac{1}{2}(a+12)$

(*i*) $\frac{5}{3}(d+1) + \frac{1}{2}d = \frac{3}{4}(d+8) + 7$ (*j*) $\frac{3}{7}y + \frac{1}{2}(y-2) = \frac{3}{4}(3y-26)$

3. (a) I think of a number x. If I divide the number by 7 and then add 10, the result equals half of the original number. Form an equation and solve it to find x.

(b) I think of a number p. If I find two-thirds of the number and subtract 4, the result is the same as halving the original number. Form an equation and solve it to find p.

(c) I think of a number n. I double this number and add on seven. If this total is then divided by three the result is one less than the original number.
Form an equation and solve it to find n.

(d) A number y is doubled and subtracted from 40. When this result is halved the answer is found to be one-and-a-half times the original number. Form an equation and solve it to find y.

4. My petrol tank hold x litres of petrol when full.
I syphon off 6 litres to put in my lawn-mower and use half of what is left on a journey.
If there are then 16 litres left, how many litres were in the full tank?

5. A roll of wire was x metres long. A 4 metre length is cut-off and used to rewire a junction box. Half of what is left is used on another job. If there is one third of the roll still unused establish the value of x.

6. A small bush, when planted, had an original height of x centimetres. After a month it had grown a further half of its original height. The height of the bush was then reduced by 21 centimetres. After this pruning the bush was found to be three quarters of its original height.

Find the original height of the bush.

Fractional Expressions

1. Simplify the following.

(a) $\dfrac{4a}{2a^2}$ (b) $\dfrac{10x^2}{12xy}$ (c) $\dfrac{3v^2t}{9vt^2}$ (d) $\dfrac{10ab^3}{2a^2b}$

(e) $\dfrac{3ab^2c}{4a^2c}$ (f) $\dfrac{4k^2m}{28km^3}$ (g) $\dfrac{5efg^2}{10e^2fg^3}$ (h) $\dfrac{21xy^2}{36x^3}$

2. Simplify the following.

(a) $\dfrac{3a}{b}\times\dfrac{b^2}{6}$ (b) $\dfrac{8b}{a}\times\dfrac{3a}{16}$ (c) $\dfrac{24x^2}{5y}\times\dfrac{10y}{6x}$ (d) $\dfrac{p^3}{24q^2}\times\dfrac{36q}{p^2}$

(e) $\dfrac{a}{2b}\div\dfrac{1}{b^2}$ (f) $\dfrac{2x}{3y}\div\dfrac{8x^2}{9}$ (g) $\dfrac{5xy}{3}\div\dfrac{10y^2}{9}$ (h) $\dfrac{6m^2}{7n}\div\dfrac{12m}{49n^2}$

(i) $\dfrac{4(a+1)}{b}\times\dfrac{b^2}{a+1}$ (j) $\dfrac{p}{3q-2}\times\dfrac{2(3q-2)}{3p}$ (k) $\dfrac{x-4}{y^2}\div\dfrac{3(x-4)}{y}$

(l) $\dfrac{2a+6}{5}\times\dfrac{10}{a+3}$ (m) $\dfrac{4r}{t+1}\times\dfrac{5t+5}{2r^2}$ (n) $\dfrac{e}{6f-12}\div\dfrac{2e^2}{3f-6}$

3. Write each of the following as a single fraction in its simplest form.

(a) $\dfrac{a}{2}+\dfrac{b}{3}$ (b) $\dfrac{k}{4}-\dfrac{m}{5}$ (c) $\dfrac{x}{3}+\dfrac{4y}{6}$ (d) $\dfrac{3h}{4}-\dfrac{k}{12}$

(e) $\dfrac{3}{a}+\dfrac{1}{b}$ (f) $\dfrac{4}{x}-\dfrac{2}{y}$ (g) $\dfrac{5}{3d}+\dfrac{1}{e}$ (h) $\dfrac{2}{5f}-\dfrac{2}{10g}$

(i) $\dfrac{7}{2b}+\dfrac{3}{4c}$ (j) $\dfrac{3}{2w}+\dfrac{8}{5x}$ (k) $\dfrac{1}{x}+\dfrac{2}{y}-\dfrac{1}{z}$ (l) $\dfrac{1}{2a}+\dfrac{2}{3b}+\dfrac{3}{c}$

4. Simplify the following.

(a) $\dfrac{x+2}{3}+\dfrac{x+3}{6}$ (b) $\dfrac{a+6}{4}+\dfrac{a-2}{3}$ (c) $\dfrac{d-3}{2}-\dfrac{d+2}{6}$

(d) $\dfrac{2a-1}{4}-\dfrac{a+2}{5}$ (e) $\dfrac{a+3b}{2}+\dfrac{a-2b}{4}$ (f) $\dfrac{2u+v}{3}-\dfrac{u-v}{4}$

(g) $\dfrac{a+2}{b}+\dfrac{a-1}{2b}$ (h) $\dfrac{3x+y}{2x}+\dfrac{x-2y}{3x}$ (i) $\dfrac{4p-q}{5p}-\dfrac{2p-3q}{3p}$

Surds (1)

1. Express each of the following in its simplest form.

(a) $\sqrt{8}$ (b) $\sqrt{12}$ (c) $\sqrt{50}$ (d) $\sqrt{20}$ (e) $\sqrt{24}$ (f) $\sqrt{108}$ (g) $\sqrt{60}$
(h) $\sqrt{72}$ (i) $\sqrt{300}$ (j) $\sqrt{27}$ (k) $\sqrt{96}$ (l) $\sqrt{48}$ (m) $\sqrt{45}$ (n) $\sqrt{98}$
(o) $5\sqrt{8}$ (p) $3\sqrt{32}$ (q) $5\sqrt{40}$ (r) $2\sqrt{12}$ (s) $4\sqrt{18}$ (t) $3\sqrt{24}$ (u) $3\sqrt{27}$

2. Simplify:

(a) $5\sqrt{2} + 3\sqrt{2}$ (b) $3\sqrt{7} - \sqrt{7}$ (c) $4\sqrt{3} + 2\sqrt{3} - 3\sqrt{3}$ (d) $5\sqrt{6} - 2\sqrt{6} + \sqrt{3}$
(e) $\sqrt{8} + 5\sqrt{2}$ (f) $5\sqrt{3} - \sqrt{12}$ (g) $4\sqrt{5} + \sqrt{20} + \sqrt{45}$ (h) $3\sqrt{2} + 2\sqrt{8} - \sqrt{18}$
(i) $\sqrt{50} - \sqrt{8}$ (j) $3\sqrt{12} + \sqrt{27}$ (k) $\sqrt{75} + \sqrt{108} - \sqrt{3}$ (l) $\sqrt{5} + \sqrt{20} + \sqrt{80}$

3. Simplify:

(a) $\sqrt{5} \times \sqrt{5}$ (b) $\sqrt{2} \times \sqrt{2}$ (c) $\sqrt{3} \times \sqrt{5}$ (d) $\sqrt{6} \times \sqrt{2}$
(e) $\sqrt{3} \times \sqrt{6}$ (f) $\sqrt{x} \times \sqrt{y}$ (g) $\sqrt{8} \times \sqrt{2}$ (h) $3\sqrt{2} \times \sqrt{2}$
(i) $2\sqrt{5} \times 3\sqrt{5}$ (j) $3\sqrt{2} \times 2\sqrt{7}$ (k) $4\sqrt{3} \times 2\sqrt{3}$ (l) $\sqrt{5} \times 3\sqrt{2}$
(m) $2\sqrt{6} \times 3\sqrt{3}$ (n) $8\sqrt{2} \times \sqrt{12}$ (o) $5\sqrt{3} \times 3\sqrt{5}$ (p) $4\sqrt{8} \times 2\sqrt{2}$

4. Expand and simplify:

(a) $\sqrt{2}(1 - \sqrt{2})$ (b) $\sqrt{3}(\sqrt{3} + 1)$ (c) $\sqrt{5}(\sqrt{5} - 1)$ (d) $\sqrt{2}(5 + \sqrt{2})$
(e) $\sqrt{2}(3 + \sqrt{6})$ (f) $2\sqrt{3}(\sqrt{8} + 1)$ (g) $\sqrt{3}(\sqrt{6} - 2\sqrt{8})$ (h) $\sqrt{5}(\sqrt{5} + 2)$
(i) $4\sqrt{6}(2\sqrt{6} - \sqrt{8})$ (j) $\sqrt{8}(\sqrt{2} + 4)$ (k) $2\sqrt{12}(\sqrt{3} + \sqrt{6})$ (l) $\sqrt{5}(\sqrt{200} + \sqrt{50})$

5. Expand and simplify:

(a) $(\sqrt{2} + 3)(\sqrt{2} - 1)$ (b) $(\sqrt{5} + 1)(2\sqrt{5} - 4)$ (c) $(2\sqrt{2} + 3)(\sqrt{2} + 4)$
(d) $(\sqrt{3} + 1)(\sqrt{3} - 1)$ (e) $(2 + \sqrt{5})(2 - \sqrt{5})$ (f) $(\sqrt{3} + \sqrt{2})(\sqrt{3} - \sqrt{2})$
(g) $(\sqrt{2} - 4)(3\sqrt{2} - 1)$ (h) $(\sqrt{8} + 2)(\sqrt{8} + 1)$ (i) $(2\sqrt{3} + \sqrt{2})(\sqrt{3} + 3\sqrt{2})$
(j) $(\sqrt{2} + 3)^2$ (k) $(\sqrt{2} + \sqrt{3})^2$ (l) $(2\sqrt{3} - 1)^2$ (m) $(2\sqrt{7} - \sqrt{2})^2$ (n) $(5 - 2\sqrt{3})^2$

6. Express each of the following with a *rational denominator*.

(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{\sqrt{3}}$ (c) $\frac{1}{\sqrt{5}}$ (d) $\frac{6}{\sqrt{3}}$ (e) $\frac{10}{\sqrt{5}}$ (f) $\frac{2}{\sqrt{3}}$ (g) $\frac{3}{\sqrt{5}}$
(h) $\frac{20}{\sqrt{2}}$ (i) $\frac{3}{2\sqrt{5}}$ (j) $\frac{4}{5\sqrt{2}}$ (k) $\frac{2}{3\sqrt{2}}$ (l) $\frac{12}{5\sqrt{6}}$ (m) $\frac{\sqrt{4}}{\sqrt{3}}$ (n) $\frac{\sqrt{5}}{\sqrt{2}}$
(o) $\frac{\sqrt{8}}{3\sqrt{2}}$ (p) $\frac{2\sqrt{3}}{3\sqrt{2}}$ (q) $\frac{1}{\sqrt{50}}$ (r) $\frac{10}{\sqrt{12}}$ (s) $\frac{4}{\sqrt{8}}$ (t) $\frac{3\sqrt{2}}{\sqrt{24}}$ (u) $\frac{2\sqrt{3}}{\sqrt{54}}$

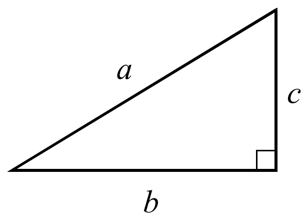
7. (extension) Rationalise the denominator, in each fraction, using the appropriate conjugate surd.

(a) $\frac{1}{\sqrt{2} - 1}$ (b) $\frac{4}{\sqrt{5} + 1}$ (c) $\frac{12}{2 - \sqrt{3}}$ (d) $\frac{2 - \sqrt{3}}{2 + \sqrt{3}}$

(e) $\frac{1}{\sqrt{3} - \sqrt{2}}$ (f) $\frac{2}{\sqrt{5} + \sqrt{3}}$ (g) $\frac{\sqrt{2}}{\sqrt{5} + 3}$ (h) $\frac{3\sqrt{2}}{\sqrt{8} + 1}$

Surds (2) - Problems

1. A right angled triangle has sides a, b and c as shown.
For each case below calculate the length of the third side, expressing your answer as a surd in its simplest form.



(a) Find a if $b = 6$ and $c = 3$. (b) Find c if $a = 2$ and $b = 1$.

(c) Find c if $a = 18$ and $b = 12$ (d) Find b if $a = 2\sqrt{8}$ and $c = 2\sqrt{6}$.

2. Given that $x = 1 + \sqrt{2}$ and $y = 1 - \sqrt{2}$, simplify:

(a) $5x + 5y$ (b) $2xy$ (c) $x^2 + y^2$ (d) $(x + y)(x - y)$

3. Given that $p = \sqrt{5} + \sqrt{3}$ and $q = \sqrt{5} - \sqrt{3}$, simplify:

(a) $2p - 2q$ (b) $4pq$ (c) $p^2 - q^2$

4. A rectangle has sides measuring $(2 + \sqrt{2})$ cm and $(2 - \sqrt{2})$ cm.

Calculate the exact value of i) its area; ii) the length of a diagonal.

5. A curve has as its equation $y = 2 + \frac{1}{2}x^2$.

(a) If the point P($\sqrt{2}$, k) lies on this curve find the exact value of k.

(b) Find the exact length of OP where O is the origin.

6. In ΔABC, AB = AC = 12cm and BC = 8cm. Express the length of the altitude from A to BC as a surd in its simplest form.

7. An equilateral triangle has each of its sides measuring $2a$ metres.

(a) Find the exact length of an altitude of the triangle in terms of a.

(b) Hence find the exact area of the triangle in terms of a.

8. The exact **area** of a rectangle is $2(\sqrt{6} + \sqrt{3})$ square centimetres. Given that the breadth of the rectangle is $\sqrt{6}$ cm, show that the length is equal to $(2 + \sqrt{2})$ cm.

9. (extension) Express $\dfrac{6}{\sqrt{7} + 1}$ in the form $a + b\sqrt{c}$.

10. (extension) Given that $\tan 75^\circ = \dfrac{\sqrt{3} + 1}{\sqrt{3} - 1}$, show that $\tan 75^\circ = 2 + \sqrt{3}$.

Indices

You must be aware of the laws of indices and all the other information shown in this box to successfully complete this worksheet

$x^a \times x^b = x^{a+b}$, $x^a \div x^b = x^{a-b}$, $(x^a)^b = x^{ab}$
$(xy)^n = x^n y^n$, $x^{-a} = \frac{1}{x^a}$, $x^{\frac{m}{n}} = \sqrt[n]{x^m}$, $x^0 = 1$

1. Simplify the following expressing your answers with positive indices.

(a) $y \times y \times y$ (b) $p \times p \times p \times p \times q \times q$ (c) $a \times a \times b \times b \times a$ (d) $2^3 \times 2^4$
(e) $p^3 \times p^7$ (f) $a^3 \times a^{-4}$ (g) $3 \times y^2 \times 2 \times y$ (h) $2 \times a^2 \times b^3 \times a^{-1}$
(i) $y^4 \div y^2$ (j) $4r^8 \div 2r^3$ (k) $d^4 \div d^{-3}$ (l) $a^0 \times 6a^3 \div 2a^{-2}$
(m) $\frac{h^5}{h^2}$ (n) $\frac{10e^{-3}}{5e^3}$ (o) $\frac{2p^{-2}}{p^{-6}}$ (p) $\frac{a^2 \times 2a^5}{a^4}$ (q) $\frac{w \times w \times w}{p^2 w^4}$ (r) $\frac{3a^4b^3}{6a^2b^5}$

2. Simplify the following expressing your answers with positive indices.

(a) $(3^2)^5$ (b) $(4^{-3})^2$ (c) $(a^3)^7$ (d) $(p^4)^{-3}$ (e) $(d^{-5})^{-2}$
(f) $(a^{\frac{1}{4}})^8$ (g) $(y^{\frac{2}{3}})^{\frac{3}{4}}$ (h) $2(p^{\frac{4}{5}})^{-\frac{5}{2}}$ (i) $3(c^{-2})^0$ (j) $(k^{-\frac{1}{3}})^{-\frac{2}{5}}$
(k) $(ab)^3$ (l) $(xy^2)^4$ (m) $(2m^3)^3$ (n) $(2xy^2)^4$ (o) $3(a^2b^4)^{\frac{1}{2}}$

3. Express without root signs.

(a) $\sqrt[4]{a}$ (b) $\sqrt{p^3}$ (c) $\sqrt[3]{x^5}$ (d) $\sqrt[5]{r^2}$ (e) $\sqrt[a]{b^3}$ (f) $\sqrt[3]{h^2}$

4. Express with root signs (write with positive indices first where necessary).

(a) $p^{\frac{1}{5}}$ (b) $w^{\frac{3}{4}}$ (c) $x^{\frac{1}{2}}$ (d) $a^{-\frac{3}{4}}$ (e) $y^{-\frac{1}{5}}$ (f) $q^{-\frac{4}{3}}$

5. Evaluate each of the following without the use of a calculator.

(a) $25^{\frac{1}{2}}$ (b) $8^{\frac{1}{3}}$ (c) $4^{-\frac{1}{2}}$ (d) $16^{-\frac{1}{4}}$ (e) 13^0 (f) 7^{-1}
(g) $16^{\frac{3}{2}}$ (h) $27^{\frac{2}{3}}$ (i) $8^{-\frac{4}{3}}$ (j) $(-8)^{\frac{1}{3}}$ (k) $64^{\frac{2}{3}}$ (l) $100^{-\frac{3}{2}}$
(m) $(\frac{1}{2})^{-1}$ (n) $(\frac{1}{8})^{\frac{4}{3}}$ (o) $(\frac{2}{3})^3$ (p) $(\frac{1}{2})^{-5}$ (q) $(\frac{3}{4})^{-2}$ (r) $(\frac{8}{27})^{-\frac{4}{3}}$

6. Simplify each of the following by …. i) changing root signs to fractional powers;
ii) moving x's onto the numerators;
iii) expanding brackets …………………… where necessary.

(a) $x^{\frac{1}{2}}(x^4 + 1)$ (b) $x^{-\frac{1}{2}}(x^{\frac{3}{2}} - x^2)$ (c) $\frac{1}{x^2}\left(x^{\frac{1}{2}} + x\right)$
(d) $\frac{2}{x^{-3}}\left(x^2 + \frac{1}{x}\right)$ (e) $\frac{1}{\sqrt{x}}\left(x^2 - \sqrt{x}\right)$ (f) $\left(x^2 + \frac{1}{x}\right)^2$
(g) $\frac{1}{x}\left(\sqrt{x} + x\right)$ (h) $\left(x + \frac{1}{\sqrt{x}}\right)^2$ (i) $x^{-2}\left(\frac{1}{x} - \sqrt[3]{x}\right)$
(j) $\frac{x^2 + 3}{x}$ (k) $\frac{\sqrt{x} - x}{x^2}$ (l) $\frac{(2x+1)^2}{x^{\frac{3}{2}}}$

Exponential Graphs

Questions 1 and 2 require 2mm graph paper.

1. W and t are linked by the formula $W = 3(2^t)$.

Copy and complete the following table of values.

t	0	1	2	3	4	5	6
W							

On 2mm graph paper, draw the graph of W against t.

2. F and p are linked by the formula $F = 80(2^{-p})$.

Copy and complete the following table of values.

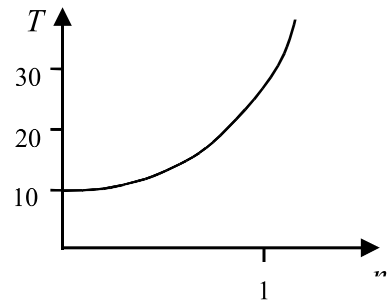
p	0	1	2	3	4
F					

On 2mm graph paper, draw the graph of F against p.

3. The graph of $T = a(b^n)$ is shown opposite.

(a) State the value of a.

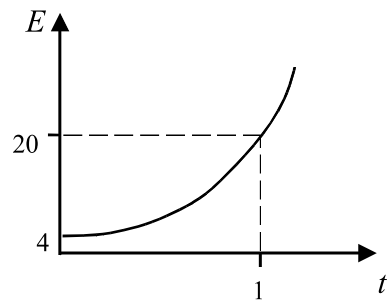
(b) Given that when $n = 2$, $T = 90$, find the value of b.



4. The graph of $E = x(y^t)$ is shown opposite.

(a) State the value of x.

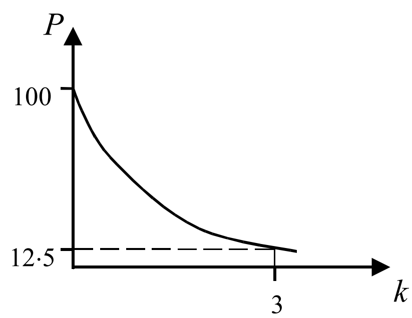
(b) Find the value of y.



5. The graph of $P = 2a(b^{-k})$ is shown opposite.

(a) State the value of a.

(b) Find the value of b.



Systems of Equations (Problems)

1. Solve the following systems of equations.

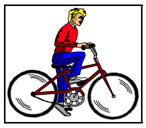
(a) $$\begin{aligned} x + y &= 5\cdot78 \\ 3x + 4y &= 20\cdot82 \end{aligned}$$

(b) $$\begin{aligned} a + b &= 8 \\ 7a - 3b &= 30 \end{aligned}$$

2. A man is paid £7.30 per hour basic and £10.40 per hour overtime.
In a particular week he worked a total of 45 hours.

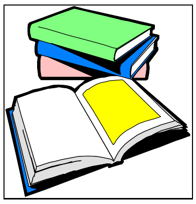
If his total wage for the week was £344, how many hours overtime did he work?

3. A shop sells two types of competition bicycle, racing and cross-country.
On a particular Saturday the shop sold a total of 82 cycles with till receipts totalling £11 245.



Given that a racing bike retails for £142 and a cross-country bike for £123, establish how many of each type of bike was sold.

4. A shop sells paper-back books for £3.90 and hard-backs for £8.80.



On a particular day 142 books were sold raising £637.10.

How many hard-back books were sold?

5. A formula for velocity (v) is given as $v = ut + \frac{1}{2}at^2$, where u is the initial velocity, a is acceleration and t is the time in seconds.

It is known that when $t = 2$, $v = 48$ and when $t = 6$, $v = 192$.

(a) Form a system of equations and solve to find u and a.

(b) Hence find v when $t = 10$.

6. A second formula for velocity (v) is $v^2 = u^2 + 2as$, where u is the initial velocity, a is acceleration and s is displacement

It is known that when $s = 20$, $v = 10$ and when $s = 90$, $v = 18$.

(a) Form a system of equations and solve to find u and a.

(b) Hence find v when $s = 200$.

Credit Maths - Mixed Exercise 1

1. Solve each of the following equations.

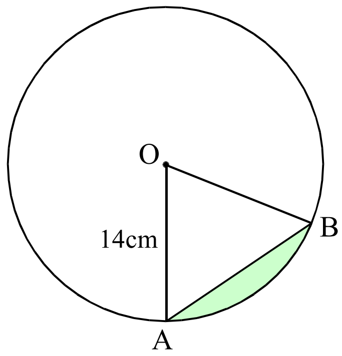
   (a) $2(3x-1)-(x+2) = 2x+17$ (b) $(2x+1)(x-6) = 2x(x-8)+4$

2. A circle centre O has a radius of 14 cm as shown.
   Chord AB has a length of 16 cm.

   (a) Calculate the size of angle AOB to the nearest degree.

   (b) Hence calculate the length of arc AB.

   (c) Calculate the area of the shaded segment.



3. The variables E, v and t are connected by the formula $E = \dfrac{kv^2}{t}$, where k is a constant.

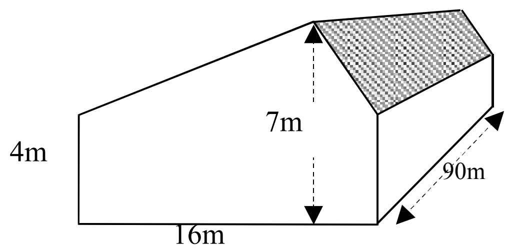
   (a) Express this relation in words by copying and completing the following variation statement
   " E varies as ………………………. and ………………….. ".

   (b) Given that $E = 200$ when $v = 50$ and $t = \frac{1}{8}$, find the value of the constant of variation k.

   Hence find E when v = 20 and t = $\frac{1}{2}$.

4. Simplify $(3a+b)^2 - (3a-b)^2$.

5. Calculate the total volume of a factory unit which is 90 metres long and whose side view dimensions are shown opposite



6. Express as a single fraction

$$\frac{x-1}{2} - \frac{x-2}{3}$$

7. Solve, correct to two decimal places, the equation $2x^2 - 4x + 1 = 0$.

8. Make x the subject of the formula $y = \sqrt{\dfrac{x-1}{x+2}}$.

Credit Maths - Mixed Exercise 2

1. (a) Solve the inequation $12 - (x - 4) > 3x$

(b) Factorise the expression $5x^2 - 19x - 4$.

Hence, i) solve the equation $5x^2 - 19x - 4 = 0$;

ii) simplify the fraction $\dfrac{x^2 - 4x}{5x^2 - 19x - 4}$.

(c) Solve the system of equations
$$4x - 3y = 27$$
$$6x + 5y = 12$$

(d) Solve the equation $\dfrac{2(x+2)}{3} - \dfrac{2x+1}{4} = \dfrac{3}{4}$

2. A sample of 12 pupils produced the following results for a survey of shoe sizes.

8 7 7 9 11 5 7 8 7 12 9 6

(a) Calculate the mean shoe size for this sample.

(b) Calculate the standard deviation.

3. A formula connected with lens physics is given as $\dfrac{1}{u} + \dfrac{1}{v} = \dfrac{1}{f}$.

(a) Make v the subject of the formula.

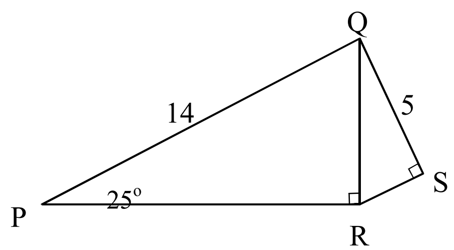
(b) Hence find v when $u = 24 \cdot 8$ and $f = 16 \cdot 8$.

4. Consider the diagram opposite.

(a) Calculate the length of QR.

(b) Hence calculate the size of angle RQS.

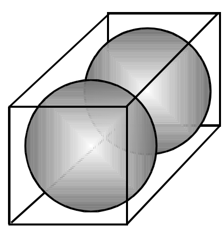
(c) Calculate the area of quadrilateral PQSR.



5. Two identical solid spheres are packed in the smallest box possible which is a cuboid in shape.

Calculate the amount of unoccupied space left in the box given that the radius of the sphere is 20cm.

[Volume of a sphere : $V = \frac{4}{3}\pi r^3$]

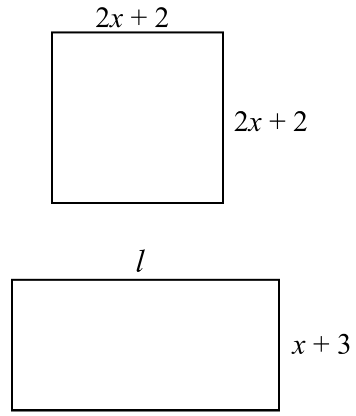


6. Solve the equation $\sqrt{5}\sin x^\circ - 1 = 0$, for $0 < x < 360$.

Credit Maths - Mixed Exercise 3

1. Solve the equation $(2x+1)^2 = 3(1-x)$.

2. A square of side $(2x+2)$ cm has the <u>same</u> perimeter as a rectangle of breadth $(x+3)$ cm and length l cm.

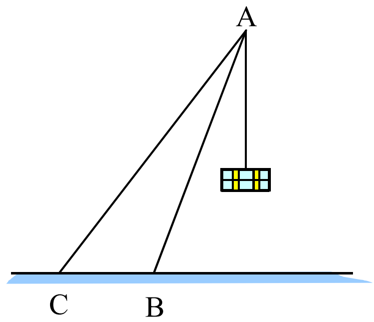


   (a) Find the length of the rectangle in terms of x.

   (b) Write down expanded expressions, in terms of x, for the areas' of the two shapes.

   (c) Given that the area of the square is $16\,\text{cm}^2$ more than the area of the rectangle, form an equation and solve it to find x.

3. The diagram opposite represents a crane for lifting weights.



   AB = 15 metres, AC = 21 metres and CB = 9 metres.

   (a) Calculate the size of angle ABC.

   (b) Calculate the vertical height of point A above the ground.

4. Given that $a = 4 \times 10^6$ and $b = 5 \times 10^{-3}$, find the value of E where $E = a \div b^2$. Write your answer in standard form.

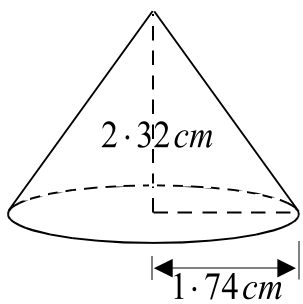
5. The cost £C of manufacturing an article is given by $C = aq^2 + \dfrac{b}{q}$, where a and b are constants.

   When q = 2, the cost of manufacture is £30 and when q = 4, the cost of manufacture is £50.

   (a) Write down two equations involving a and b and solve them simultaneously to find a and b.

   (b) Hence calculate the cost of manufacture when q = 10.

6. A solid cone has a base radius of $1 \cdot 74$ cm and a vertical height of $2 \cdot 32$ cm. Calculate the cone's



   i) slant height;
   ii) total surface area;
   iii) volume.

   Give all answers correct to 3 significant figures. [Surface Area = $\pi r^2 + \pi rs$; $V = \frac{1}{3}\pi r^2 h$]

7. A bicycle wheel of diameter 60cm makes 5 revolutions every 2 seconds.

   Find the speed of the bicycle to the nearest km/h.

Credit Maths - Mixed Exercise 4

1. Solve i) $1 + \frac{1-4x}{3} = \frac{3(x-1)}{5}$

ii) $(x+6)^2 = 4(5x+6)$

2. A cheese spread comes in a cuboid shaped box measuring 4cm by 8cm by 10cm. If the box is to be changed to a cylindrical one of depth 4cm what should be the diameter of the new box if the volume of the contents is to remain the same?

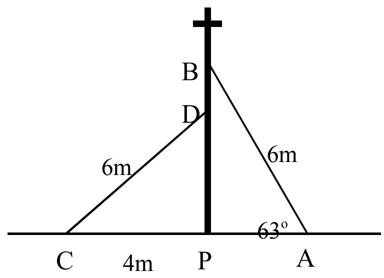
3. The sag (S mm) at the centre of a beam varies as the fourth power of its length (l m) and inversely as the square of its depth (d cm).

If a beam 10 metres long and 5cm deep sags 2mm, find the sag of a beam 12 metres long and 6cm deep.

4. The diagram represents a vertical telegraph pole which is supported by two straight ropes AB and CD of length 6 metres.

CP is 4 metres and angle PAB = 63°.

Calculate the distance between the points B and D giving your answer in centimetres to the nearest centimetre.



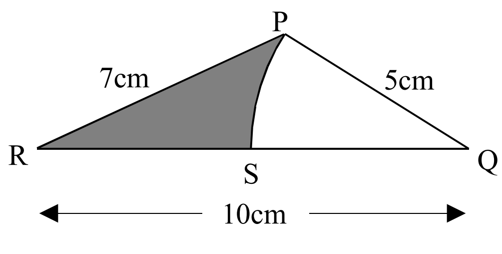
5. Find,correct to two decimal places, the roots of the equation $5x^2 - 3x - 1 = 0$.

6. A contractor estimated that he needed 48 men to complete a job in 14 days, but he has been asked to complete the work in 12 days.

How many more men will he have to hire, assuming the same rate of working?

7. Triangle PQR has sides PQ = 5cm, QR = 10cm and RP = 7cm as shown. PS is an arc of a circle with centre Q.

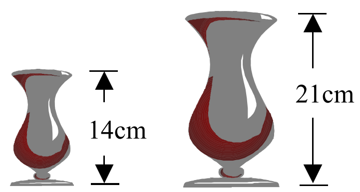
Calculate the shaded area.



8. The flower vases opposite are mathematically similar.

To fill the small vase with water takes 400 millilitres.

How much water, **in litres**, is needed to fill the large vase?



9. Simplify: (a) $\frac{3x^2 - 6x}{x^2 - 4}$ (b) $\frac{4a^2 - 9}{2a^2 + 5a - 12}$

Answers to S4 Credit Revision Pack

Gradient & Equation of a Line

1. (a) (i) $m = 2$ (ii) $m = -\frac{1}{2}$ (iii) $m = \frac{4}{3}$ (iv) $m = -\frac{1}{6}$ (v) $m = 1$

   (b) *zero* (0) , *undefined*

2. (a) $m = 2$ (b) $m = -4$ (c) $m = \frac{1}{5}$ (d) $m = -2$ (e) $m = 1$

   (f) $m = -\frac{3}{4}$ (g) $m = \frac{5}{3}$ (h) $m = -\frac{2}{5}$ (i) $m = -\frac{11}{7}$

3. Students own drawings.

4. (a) $m = 1$, (0,3) (b) $m = -2$, (0,-1) (c) $m = \frac{1}{2}$, (0,0)

   (d) $m = -\frac{1}{2}$, (0,2) (e) $m = -1$, (0,6) (f) $m = \frac{1}{2}$, (0,-2)

   (g) $m = \frac{1}{3}$, (0,4) (h) $m = -\frac{4}{5}$, (0,4) (i) $m = \frac{3}{2}$, (0,-6)

5. (a) $m = 1$, (0,-7) (b) $m = -5$, (0,3) (c) $m = \frac{3}{5}$, (0,-2)

   (d) $m = -4$, (0,0) (e) $m = -2$, (0,11) (f) $m = \frac{1}{2}$, $(0, -\frac{5}{2})$

   (g) $m = \frac{1}{3}$, (0,6) (h) $m = -\frac{3}{7}$, (0,3) (i) $m = \frac{4}{5}$, (0,-4)

6. (a) $y = 4x + 5$ (b) $y = -2x + 1$ (c) $y = \frac{3}{4}x - 3$

7. (a) $y = 3x - 5$ (b) $y = -4x + 16$ (c) $y = \frac{1}{2}x$

   (d) $y = -2x - 5$ (e) $y = \frac{1}{2}x + 7$ (f) $y = -\frac{1}{3}x + 1$

8. (a) $y = 2x - 5$ (b) $y = -4x + 13$ (c) $5y = x + 32$

   (d) $y = -2x + 1$ (e) $y = x + 6$ (f) $4y = -3x - 15$

   (g) $3y = 5x + 20$ (h) $5y = -2x + 14$ (i) $7y = -11x + 48$

Functions & Graphs (1)

1. (a) 13 (b) -11 (c) -2 (d) $6a-5$
2. (a) 8 (b) 20 (c) 13 (d) p^2+4
3. (a) 4 (b) 0 (c) 16 (d) $12-2m$
4. (a) a^2+3a (b) $4p^2+6p$ (c) m^2+5m+4 (d) $e^2-7e+10$
5. (a) 0 (b) $9a^2-12a$ (c) $a^2-8a+12$ (d) $4p^2-4p-3$
6. (a) $x=4$ (b) $x=-1$ (c) $x=0\cdot 4$ *or* $\frac{2}{5}$
7. (a) $t=3$ (b) $t=6$ (c) $t=-2$
8. (a) $a=\pm 5$ (b) $a=\pm 1$ (c) $a=\pm 4$
9. (a) i) 15 ii) 0 (b) $a^2+8a+15$
10. (a) i) 9 ii) 39 (b) $t=5\cdot 5$ $(5\frac{1}{2})$ (c) $45-6p$

Functions & Graphs (2)

1. (a)

x	-2	-1	0	1	2	3	4
$f(x)$	5	0	-3	-4	-3	0	5

(b) student's graph (c) minimum , (1,-4) (d) $x=-1$ *or* $x=3$

2. (a)

x	0	1	2	3	4	5	6	7	8
$g(x)$	12	5	0	-3	-4	-3	0	5	12

(b) studen'ts graph (c) minimum , (4,-4) (d) $x=2$ *or* $x=6$

3. (a)

x	-1	0	1	2	3	4	5	6	7	8	9
$h(x)$	-9	0	7	12	15	16	15	12	7	0	-9

(b) student's graph (c) maximum , (4,16) (d) $x=0$ *or* $x=8$

4. (a)

x	-1	0	1	2	3	4
$f(x)$	4	0	2	4	0	-16

(b) student's graph (c) min , (0,0) : max , (2,4) (d) $x = 0$ *or* $x = 3$

The Circle (1)

1. (a) 12·6 cm (b) 34·2mm (c) 1·2m
2. (a) 28·6cm b) 62·2mm (c) 5·2m
3. (a) 50·2cm^2 (b) 239·3mm^2 (c) 1·2m^2
4. (a) 271·6m (b) 4465·8m^2
5. (a) 83·4cm (b) 150·7cm^2
6. (a) 1317·5cm^2 (b) 176·7cm

The Circle (2)

1. (a) 9cm^2 (b) 96mm^2 (c) 2·3cm^2
2. (a) 8·1mm (b) 43·5cm (c) 2·5cm
3. (a) 17mm (b) 97·9cm (c) 5·2cm
4. (a) 13cm (b) 2·4cm^2
5. (a) 213cm (b) 2541cm^2

The Circle (3)

1. (a) 29·4cm^2 (b) 90^o
2. 20^o
3. (a) 57·3^o (b) 89·6^o
   (c) 12·2cm (d) 4·8cm^2
4. 25·1m^2

The Circle (4)

Exercise 1

1. 33^o 2. 28^o 3. 138^o

4. 86^o 5. 49^o, 49^o` 6. 40^o

7. 61°   8. 36°, 54°

Exercise 2

1. 30°   2. 65°

3. 50°   4. 45°

Inequations

1. (a) {-2, -1, 0, 1}   (b) {2, 3, 4, 5}

   (c) {5}   (d) {-2, -1}

   (e) {-2, -1, 0}   (f) {1, 2, 3, 4, 5}

   (g) {-2, -1, 0, 1, 2}   (h) {1, 2, 3, 4, 5}

2. (a) $a \leq 3$ (b) $x > 1$

   (c) $p \geq 2$ (d) $k < -3$

   (e) $m \leq \frac{23}{3}$ $(7\frac{2}{3})$ (f) $y > -\frac{29}{10}$ $(-2\frac{9}{10})$

   (g) $h < \frac{6}{7}$ (h) $x > \frac{1}{5}$

3. (a) $a \geq 3$ (b) $x < 2$

   (c) $p \leq 2$ (d) $k > -2$

   (e) $d \geq \frac{38}{9}$ $(4\frac{2}{9})$ (f) $y < -\frac{10}{3}$ $(-3\frac{1}{3})$

   (g) $h > 0$ (h) $y < \frac{11}{9}$ $(1\frac{2}{9})$

   (i) $w < \frac{5}{4}$ - $(-1\frac{1}{4})$ (j) $d > 10$

4. {0, 1, 2, 3, 4, 5}

5. {0,1,2}

6. Jane must be younger than 11 years.

Trigonometry (1)

1.	(a)	11·3cm	(b)	35·3°				
2.	(a)	13·4cm, 10cm	(b)	36·7°				
3.	(a)	36·9°	(b)	10m, 22·6°				
4.	(a)	23·3cm	(b)	59°	(c)	(i)	17cm	
						(ii)	68·6°	
						(iii)	184·4cm^2	

Trigonometry (2)

1.	(a)	10·6cm	(b)	26·2cm	(c)	14·1cm
	(d)	3·5cm	(e)	1·2m	(f)	2·8cm
	(g)	3mm	(h)	4·5m		
2.	(a)	28°	(b)	32·6°	(c)	41°
	(d)	122·3°	(e)	115·4°	(f)	79·9°

Trigonometry (3)

1.	(a)	3·2cm	(b)	3cm	(c)	4·9mm
	(d)	15·5cm	(e)	4m	(f)	2·9cm
	(g)	16·2mm	(h)	45·9m		
2.	(a)	34·9°	(b)	26·9°	(c)	40·8°
	(d)	96·4°	(e)	119·9°	(f)	77·4°

Trigonometry (4)

1.	(a)	19·3cm^2	(b)	61·4m^2	(c)	23·6mm^2
	(d)	2.7cm^2	(e)	298·8cm^2	(f)	119·4cm^2
2.	(a)	311·8cm^2	(b)	75·8cm^2		
3.	119m^2					

Trigonometry (5)

!. (a) 8·5 (b) 6·3 (c) 26°

(d) $75{\cdot}5^\circ$

2. $82{\cdot}6\text{cm}^2$

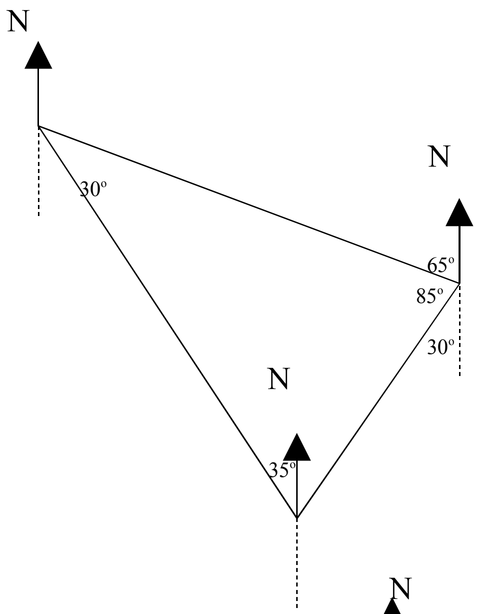
3. (a) 11·8 (b) 8·2 (c) 110

4. (a) 6·7cm (b) 48° (c) 100°

(d) 8·9cm (e) $25{\cdot}8\text{cm}^2$

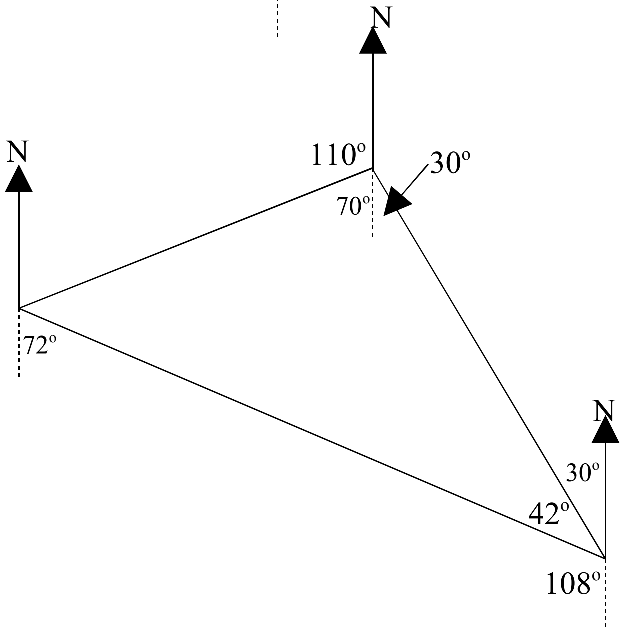
Trigonometry (6)

1. (a)



(b)
(i) 115°
(ii) 295°
(iii) 210°
(iv) 325°
(v) 145°
(vi) 030°

2. (a)



(i) 070°
(ii) 250°
(iii) 150°
(iv) 288°
(v) 108°
(vi) 330°

Trigonometry (6)……. cont

3. (a) 278km (b) 276^o

4. 4·77km

5. 108·2m

6. 16·2km

Trigonometry (7)

1.	(a)	$70{\cdot}5^o$, $289{\cdot}5^o$	(b)	210^o, 330^o	(c)	$60{\cdot}3^o$, $240{\cdot}3^o$
	(d)	$101{\cdot}5^o$, $258{\cdot}5^o$	(e)	$14{\cdot}5^o$, $165{\cdot}5^o$	(f)	$158{\cdot}2^o$, $338{\cdot}2^o$
	(g)	$45{\cdot}6^o$, $134{\cdot}4^o$	(h)	45^o, 315^o	(i)	240^o, 300^o
	(j)	$62{\cdot}2^o$, $242{\cdot}2^o$	(k)	0^o, 180^o	(l)	$93{\cdot}1^o$, $346{\cdot}9^o$
	(m)	$68{\cdot}1^o$, $248{\cdot}1^o$	(n)	$6{\cdot}4^o$, $73{\cdot}6^o$		
2.	(a)	60^o, 300^o	(b)	$23{\cdot}6^o$, $156{\cdot}4^o$	(c)	$126{\cdot}9^o$, $233{\cdot}1^o$
	(d)	45^o, 215^o	(e)	120^o, 240^o	(f)	$53{\cdot}1^o$, 126.9^o
	(g)	$118{\cdot}2^o$, $241{\cdot}8^o$	(h)	$145{\cdot}5^o$, $354{\cdot}5^o$	(i)	$116{\cdot}6^o$, $273{\cdot}4^o$
	(j)	$1{\cdot}9^o$, $108{\cdot}1^o$				

3. (a) $19{\cdot}5^o$, $160{\cdot}5^o$, $199{\cdot}5^o$, $340{\cdot}5^o$

   (b) $54{\cdot}7^o$, $125{\cdot}3^o$, $234{\cdot}7^o$, $305{\cdot}3^o$

   (c) $63{\cdot}4^o$, $116{\cdot}6^o$, $243{\cdot}4^o$, $296{\cdot}6^o$

   (d) $40{\cdot}9^o$, $139{\cdot}1^o$, $220{\cdot}9^o$, $319{\cdot}1^o$

Solving Quadratic Equations (1)

Exercise 1

(a)	-6, 0	(b)	0, 9	(c)	0, 1	(d)	-7, 0
(e)	-11, 0	(f)	0, 3	(g)	0, 7	(h)	0, 12
(i)	-5, 0	(j)	0, 6	(k)	$0, \frac{5}{2}$	(l)	$0, \frac{7}{2}$
(m)	$-\frac{5}{3}, 0$	(n)	$-\frac{4}{3}, 0\cdot$	(o)	$-\frac{7}{6}, 0$	(p)	$0, \frac{5}{4}$
(q)	$0, \frac{9}{8}$	(r)	$-\frac{16}{9}, 0$	(s)	$0, \frac{5}{4}$	(t)	$0, \frac{9}{2}$

Exercise 2

(a)	-6, 6	(b)	-10, 10	(c)	-7, 7	(d)	-1, 1
(e)	-8, 8	(f)	-11, 11	(g)	$-\frac{5}{2}, \frac{5}{2}$	(h)	$-\frac{4}{3}, \frac{4}{3}$
(i)	$-\frac{8}{5}, \frac{8}{5}$	(j)	$-\frac{5}{3}, \frac{5}{3}$	(k)	$-\frac{1}{5}, \frac{1}{5}$	(l)	$-\frac{3}{2}, \frac{3}{2}$
(m)	$-\frac{12}{7}, \frac{12}{7}$	(n)	$-\frac{5}{6}, \frac{5}{6}$	(o)	$-\frac{1}{9}, \frac{1}{9}$	(p)	$-\frac{1}{2}, \frac{1}{2}$
(q)	$-\frac{3}{4}, \frac{3}{4}$	(r)	$-\frac{1}{4}, \frac{1}{4}$	(s)	$-\frac{5}{2}, \frac{5}{2}$	(t)	-3, 1

Exercise 3

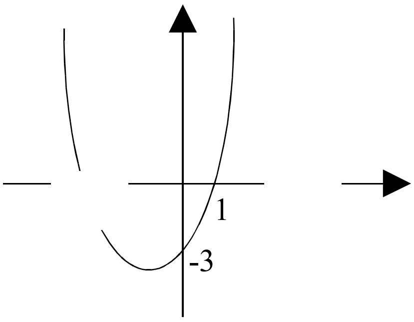
(a)	4, 5	(b)	3, 9	(c)	1, 5	(d)	3, 5
(e)	-10, -4	(f)	-9, -1	(g)	-9 (twice)	(h)	-8, 6
(i)	-12, 5	(j)	-9, 4	(k)	-8, 7	(l)	-3, 7
(m)	-6, 12	(n)	-1, 8	(o)	$-\frac{7}{3}, -1$	(p)	$\frac{1}{2}, 5$
(q)	$\frac{1}{4}, \frac{7}{3}$	(r)	$\frac{7}{3}, 4$	(s)	$\frac{5}{2}$ (twice)	(t)	$-\frac{4}{3}, -\frac{2}{3}$
(u)	$-5, \frac{1}{3}$	(v)	$-\frac{3}{2}, \frac{4}{3}$	(w)	$-\frac{5}{4}, \frac{3}{2}$	(x)	$-2, \frac{9}{2}$
(y)	$-\frac{5}{6}, \frac{3}{2}$	(z)	$-\frac{7}{2}, 4$				

Solving Quadratic Equations (2)

1.	(a)	-5, 1	(b)	-5, -1	(c)	-3, 4
	(d)	-4, 7	(e)	1, 9	(f)	5, -3
	(g)	-3, 3	(h)	-1, 1	(i)	-3, $-\frac{1}{2}$
	(j)	-1, $-\frac{2}{3}$	(k)	-3, 3	(l)	$-\frac{2}{3}$, $\frac{1}{2}$
	(m)	-6, 12	(n)	$-\frac{3}{2}$, $\frac{3}{2}$	(o)	$-\frac{7}{3}$, -1
	(p)	$-\frac{5}{2}$, 2	(q)	$-\frac{2}{3}$, $\frac{2}{3}$	(r)	$\frac{7}{3}$, 4
	(s)	$-\frac{2}{3}$ (twice)	(t)	-2, $\frac{3}{4}$	(u)	$\frac{1}{4}$, $\frac{2}{3}$
	(v)	$-\frac{3}{2}$, $\frac{4}{3}$	(w)	$\frac{2}{3}$, $\frac{3}{2}$	(x)	$-\frac{1}{5}$, $\frac{1}{5}$
	(y)	$-\frac{2}{5}$, $\frac{3}{4}$	(z)	$-\frac{1}{2}$, $\frac{4}{3}$		
2.	(a)	-3·7, -0·3	(b)	-5·2, -0·8	(c)	-6·2, -0·8
	(d)	-3·3, 0·3	(e)	0·6, 5·4	(f)	-1·3, 5·8
	(g)	-1·7, 1·2	(h)	-0·9, 2·3	(i)	-0·9, -5·1
	(j)	0·3, 2·7	(k)	-1·5, 1·1	(l)	-0·6, 0·2
3.	(a)	-0·59, -3·4	(b)	-6·32, 0·32	(c)	-2·41, 0·41
	(d)	-7·61, 0·39	(e)	-0·61, 6·61	(f)	2·13, 9·87
	(g)	-2·58, 0·58	(h)	0·78, 3·22	(i)	-5·37, 0·37
4.	(a)	-5,2	(b)	-0·65, 4·65	(c)	-0·45, 4·45
	(d)	-2, 3	(e)	-2, 3	(f)	-1, 2

Quadratic Graphs

1. (a)



(i) -3, 1

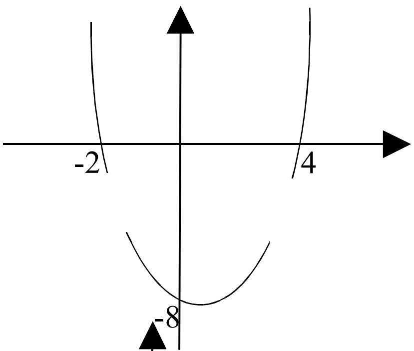
(ii) $x = -1$

(iii) (-1, -4), minimum

(vi) (0, -3)

(v) $y \geq -4$

(b)



(i) -2, 4

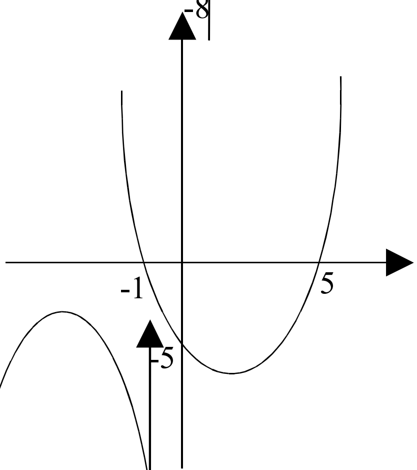
(ii) $x = 1$

(iii) (1, -9), minimum

(iv) (0, -8)

(v) $y \geq -9$

(c)



(i) -1, 5

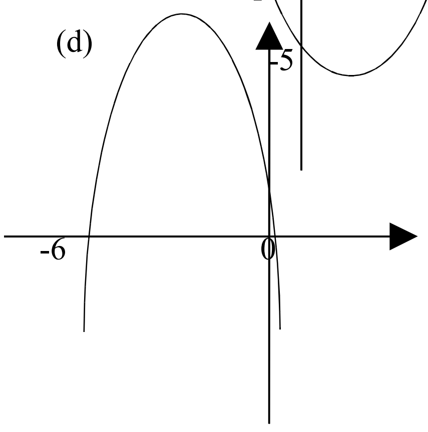
(ii) $x = 2$

(iii) (2, -9), minimum

(iv) (0, -5)

(v) $y \geq -9$

(d)



(i) -6, 0

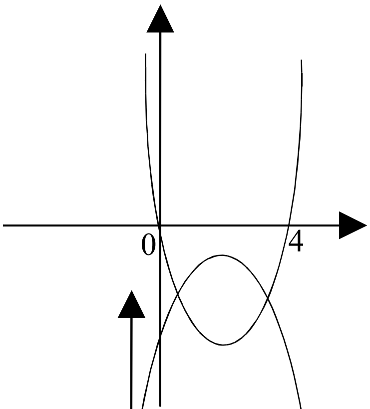
(ii) $x = -3$

(iii) (-3, -9), minimum

(iv) (0,0)

(v) $y \geq -9$

(e)



(i) 0, 4

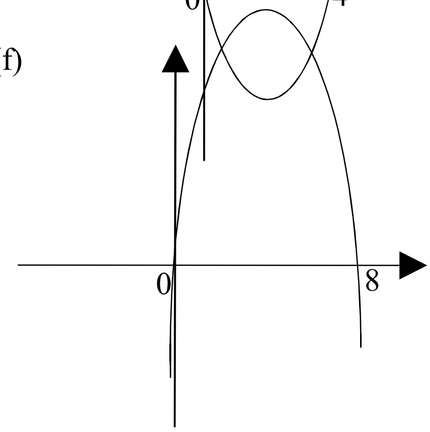
(ii) $x = 2$

(iii) (2, -4), minimum

(iv) (0,0)

(v) $y \geq -4$

(f)



(i) 0, 8

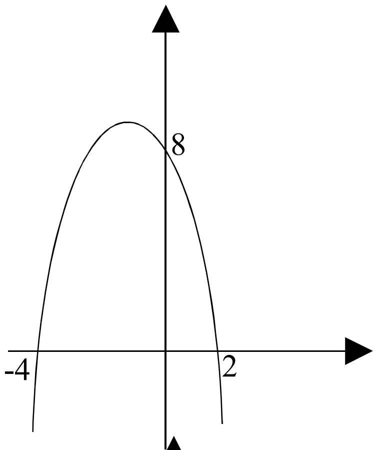
(ii) $x = 4$

(iii) (4, 16), maximum

(iv) (0,0)

(v) $y \leq 16$

(g)



(i) -4, 2

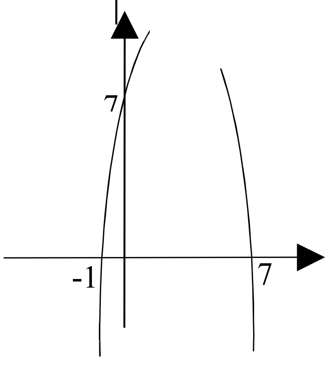
(ii) $x = -1$

(iii) (-1, 9), maximum

(iv) (0,8)

(v) $y \leq 9$

(h)



(i) -1, 7

(ii) $x = 3$

(iii) (3,16), maximum

(iv) (0,7)

(v) $y \leq 16$

(i)

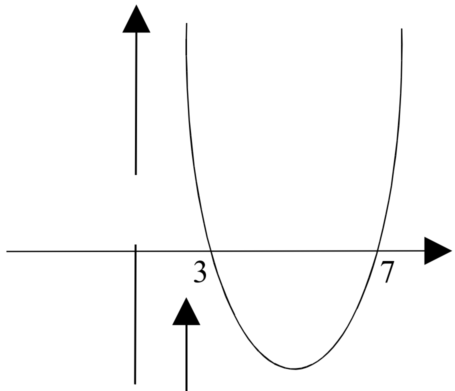
(i) 3, 7

(ii) $x = 5$

(iii) (5,-4), minimum

(iv) (0,21)

(v) $y \geq -4$



(j)

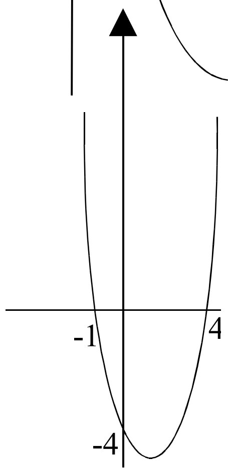
(i) -1, 4

(ii) $x = 1\cdot5$

(iii) (1·5, -6·25), minimum

(iv) (0,-4)

(v) $y \geq -6\cdot25$



(k)

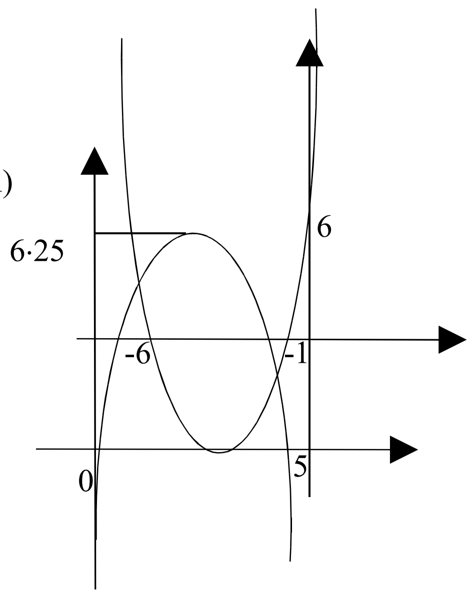
(i) -1, -6

(ii) $x = -3\cdot5$

(iii) (-3·5,-6·25), minimum

(iv) (0,6)

(v) $y \geq -6\cdot25$



(l)

(i) 0, 5

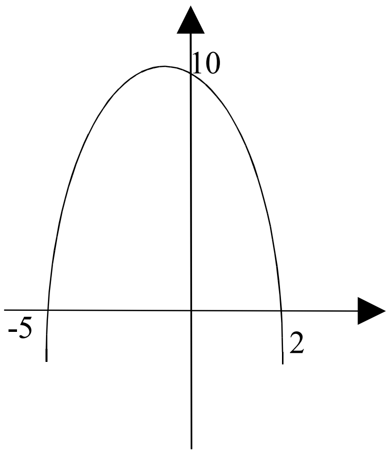
(ii) $x = 2\cdot5$

(iii) (2·5,6·25), maximum

(iv) (0,0)

(v) $y \leq 6\cdot25$

(m)



(i) -5, 2

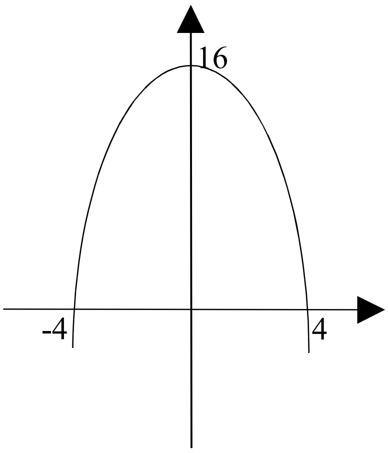
(ii) $x = -1 \cdot 5$

(iii) $(-1 \cdot 5, 12 \cdot 25)$, maximum

(iv) (0,10)

(v) $y \leq 12 \cdot 25$

(n)



(i) -4, 4

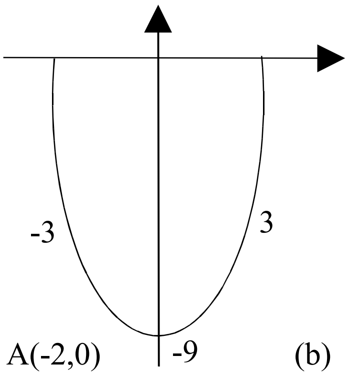
(ii) $x = 0$

(iii) (0,16), maximum

(iv) (0,16)

(v) $y \leq 16$

(o)



(i) -3, 3

(ii) $x = 0$

(iii) (0,-9), minimum

(iv) (0,-9)

(v) $y \geq -9$

2. (a) A(-2,0)
B(8,0)
C(3,-25)
D(0,-16)

(b) E(-11,0)
F(3,0)
G(-4,-49)
H(0,-33)

(c) I(-6,0)
J(-2,0)
K(-4,-4)
L(0,12)

(d) M(-6,0)
N(4,0)
P(-1,-25)
Q(0,-24)

(e) R(-1,0)
S(5,0)
T(2, -9)
U(0,-5)

(f) A(1,0)
B(7,0)
C(4,-9)
D(0,7)

Establishing the Equation of a Quadratic Graph

1. (a) $x^2 - 6x - 7$ (b) $2x^2 - 12x - 32$ (c) $3x^2 - 30x + 27$

2. (a) $480 + 28p - 2p^2$ (b) $\frac{1}{2}x^2 - 6x - 80$ (c) $4x - \frac{1}{10}x^2$

3. $f(x) = 10x - \frac{1}{5}x^2$

4. $R = 0{\cdot}6t - 0{\cdot}01t^2$ (a) 4·69 cl/min (b) 40 cl/min

Problems Involving Quadratic Equations

1. (a) $(x + 7)^2 + x^2 = 13^2$, 5 (b) 34cm

2. 9 54cm^2

3. 10 120cm^2

4. (a) $80 - x$ (b) $x(80 - x) = 1500$ 30, 50 Dimensions are 30m x 50m

5. (a) $130 - x$ (b) $x(130 - x) = 4000$ 80m/50m

6. 5m, 16m

7. (a) $2{\cdot}5 - d$ (b) proof (c) 0·5

Iteration

1. 1·3 2. 1·8 3. -0·5 4. (b) 1·4

5. (b) 3·8 6. (b) -3·5 7. (b) 1·2

Standard Deviation

1. (a) 32, 12 (b) Mean of second sample is slightly lower but the heights are closer together.

2. (a) 4·8, 0·19 (b) Second sample is less consistent than the first.

3. (a) 38, 7·22 (b) 1

4. (a) 62·4, 1·89 (b) 2

Variation (1)

Direct Variation

1. (a) $C \propto t$, $C = kt$ (b) $V \propto r^2$, $V = kr^2$ (c) $T \propto \sqrt{r}$, $T = k\sqrt{r}$ (d) $V \propto r^3$, $V = kr^3$

2. 2, 10

3. 108

4. £7500

5. 1.6 seconds

6. 56 cm

Inverse Variation

1. (a) $N \propto \frac{1}{p}$, $N = \frac{k}{p}$ (b) $V \propto \frac{1}{\sqrt{h}}$, $V = \frac{k}{\sqrt{h}}$ (c) $e \propto \frac{1}{d^2}$, $e = \frac{k}{d^2}$

2. 2

3. 10 km/h

4. 0·52 seconds

Variation (2) - Joint Variation

1. (a) $E \propto \frac{t}{p^2}$, $E = \frac{k}{p^2}$ (b) $Q \propto h^2t^3$, $Q = kh^2t^3$ (c) $M \propto \frac{eg^2}{\sqrt{d}}$, $M = \frac{keg^2}{\sqrt{d}}$ (d) $T \propto \frac{mpg}{s^2}$, $V = \frac{k\,mpg}{s^2}$

   (e) $V \propto hr^2$, $V = khr^2$ (f) $T \propto \frac{\sqrt{l}}{g^2}$, $T = \frac{k\sqrt{l}}{g^2}$

2. 48

3. 15

4. 10

5. (a) $e = \frac{32f}{g^2}$ (b) 64

6. 2700 lb.wt.

7. 6 men

8. 157.5

9. (a) proof
   (b) 33 cm^3

Equations Involving Fractions (1)

1. (a) 24 (b) 4 (c) 12 (d) 15
   (e) 6 (f) $\frac{21}{8}$ (g) $\frac{4}{5}$ (h) $\frac{12}{35}$
   (i) 5 (j) 3 (k) $\frac{4}{3}$ (l) 16
   (m) 2 (n) 12 (o) 10 (p) $\frac{45}{8}$
2. (a) 16 (b) 2 (c) 6
   (d) 8 (e) 15 (f) 10
   (g) 16 (h) 24 (i) 6
   (j) $\frac{20}{3}$ (k) $\frac{20}{7}$ (l) $\frac{5}{4}$
   (m) $\frac{40}{9}$ (n) $\frac{6}{5}$ (o) $\frac{30}{7}$
   (p) $\frac{7}{5}$ (q) -12 (r) 3
   (s) -16 (t) 4 (u) $\frac{15}{4}$
3. (a) 3 (b) 1 (c) 6
   (d) $\frac{1}{2}$ (e) -4 (f) 2
   (g) -2 (h) $\frac{1}{3}$ (i) $\frac{9}{5}$

Equations Involving Fractions (2)

1. (a) 20 (b) 6 (c) 14
   (d) $\frac{-1}{16}$ (e) $\frac{-11}{18}$ (f) $\frac{47}{48}$
   (g) $\frac{-1}{12}$ (h) $\frac{25}{36}$ (i) $\frac{124}{45}$
2. (a) 3 (b) 6 (c) 7
   (d) 10 (e) 7 (f) $\frac{59}{5}$
   (g) -1 (h) 12 (i) 8 (j) 14
3. (a) 28 (b) 24 (c) 10 (d) 8
4. 38 litres
5. 12
6. 28 cm.

Fractional Expressions

1. (a) $\frac{2}{a}$ (b) $\frac{5x}{6y}$ (c) $\frac{v}{3t}$ (d) $\frac{5b^2}{a}$

   (e) $\frac{3b^2}{4a}$ (f) $\frac{k}{7m^2}$ (g) $\frac{1}{2eg}$ (h) $\frac{7y^2}{12x^2}$

2. (a) $\frac{ab}{2}$ (b) $\frac{3b}{2}$ (c) $8x$ (d) $\frac{3p}{2q}$

   (e) $\frac{ab}{2}$ (f) $\frac{3}{4xy}$ (g) $\frac{3x}{5y}$ (h) $\frac{7mn}{2}$

   (i) $4b$ (j) $\frac{2}{3}$ (k) $\frac{1}{3y}$ (l) 4

   (m) $\frac{10}{r}$ (n) $\frac{1}{4e}$

3. (a) $\frac{3a+2b}{6}$ (b) $\frac{5k-4m}{20}$ (c) $\frac{x+2y}{3}$ (d) $\frac{9h-k}{12}$

   (e) $\frac{3b+a}{ab}$ (f) $\frac{4y-2x}{xy}$ (g) $\frac{5e+3d}{3de}$ (h) $\frac{2g-f}{5fg}$

   (i) $\frac{14c+3b}{4bc}$ (j) $\frac{15x+16w}{10xw}$ (k) $\frac{yz+2xz-xy}{xyz}$ (l) $\frac{3bc+4ac+18ab}{6abc}$

4. (a) $\frac{3x+7}{6}$ (b) $\frac{7a+10}{12}$ (c) $\frac{2d-11}{6}$ (d) $\frac{6a-13}{12}$

   (e) $\frac{3a+4b}{4}$ (f) $\frac{5u+7v}{12}$ (g) $\frac{3a+3}{2b}$ (h) $\frac{11x-y}{6x}$

   (i) $\frac{2p+12q}{15p}$

Surds(1)

1. (a) $2\sqrt{2}$ (b) $2\sqrt{3}$ (c) $5\sqrt{2}$ (d) $2\sqrt{5}$ (e) $2\sqrt{6}$ (f) $6\sqrt{3}$

   (g) $2\sqrt{15}$ (h) $6\sqrt{2}$ (i) $10\sqrt{3}$ (j) $3\sqrt{3}$ (k) $4\sqrt{6}$ (l) $4\sqrt{3}$

   (m) $3\sqrt{5}$ (n) $7\sqrt{2}$ (o) $10\sqrt{2}$ (p) $12\sqrt{2}$ (q) $10\sqrt{10}$ (r) $4\sqrt{3}$

   (s) $12\sqrt{2}$ (t) $6\sqrt{6}$ (u) $9\sqrt{3}$

2. (a) $8\sqrt{2}$ (b) $2\sqrt{7}$ (c) $3\sqrt{3}$ (d) $4\sqrt{6}$ (e) $7\sqrt{2}$ (f) $3\sqrt{3}$

   (g) $9\sqrt{5}$ (h) $4\sqrt{2}$ (i) $3\sqrt{2}$ (j) $9\sqrt{3}$ (k) $10\sqrt{3}$ (l) $7\sqrt{5}$

3. (a) 5 (b) 2 (c) $\sqrt{15}$ (d) $2\sqrt{3}$ (e) $3\sqrt{2}$ (f) $\sqrt{(xy)}$

   (g) 4 (h) 6 (i) 30 (j) $6\sqrt{14}$ (k) 24 (l) $3\sqrt{10}$

   (m) $18\sqrt{2}$ (n) $16\sqrt{6}$ (o) $15\sqrt{15}$ (p) 32

Surds(1) ….cont

4.	(a)	$\sqrt{2} - 2$	(b)	$3 + \sqrt{3}$	(c)	$5 - \sqrt{5}$	(d)	$5\sqrt{2} + 2$
	(e)	$3\sqrt{2} + 2\sqrt{3}$	(f)	$4\sqrt{6} + 2\sqrt{3}$	(g)	$3\sqrt{2} - 4\sqrt{6}$	(h)	$5 + 2\sqrt{5}$
	(i)	$48 - 16\sqrt{3}$	(j)	$4 + 8\sqrt{2}$	(k)	$12 + 12\sqrt{2}$	(l)	$15\sqrt{10}$
5.	(a)	$2\sqrt{2} - 1$	(b)	$6 - 2\sqrt{5}$	(c)	$11\sqrt{2} + 16$	(d)	2
	(e)	-1	(f)	1	(g)	$10 - 13\sqrt{2}$	(h)	$10 + 3\sqrt{8}$
	(i)	$12 + 7\sqrt{6}$	(j)	$11 + 6\sqrt{2}$	(k)	$5 + 2\sqrt{6}$	(l)	$13 - 4\sqrt{3}$
	(m)	$30 - 4\sqrt{14}$	(n)	$37 - 20\sqrt{3}$				
6.	(a)	$\frac{\sqrt{2}}{2}$	(b)	$\frac{\sqrt{3}}{3}$	(c)	$\frac{\sqrt{5}}{5}$	(d)	$2\sqrt{3}$
	(e)	$2\sqrt{5}$	(f)	$\frac{2\sqrt{3}}{3}$	(g)	$\frac{3\sqrt{5}}{5}$	(h)	$10\sqrt{2}$
	(i)	$\frac{3\sqrt{5}}{10}$	(j)	$\frac{2\sqrt{2}}{5}$	(k)	$\frac{\sqrt{2}}{3}$	(l)	$\frac{2\sqrt{6}}{5}$
	(m)	$\frac{2\sqrt{3}}{3}$	(n)	$\frac{\sqrt{10}}{2}$	(o)	$\frac{2}{3}$	(p)	$\frac{\sqrt{6}}{3}$
	(q)	$\frac{\sqrt{2}}{10}$	(r)	$\frac{5\sqrt{3}}{3}$	(s)	$\sqrt{2}$	(t)	$\frac{\sqrt{3}}{2}$ (u) $\frac{\sqrt{2}}{3}$
7.	(a)	$\sqrt{2} + 1$	(b)	$\sqrt{5} - 1$	(c)	$12(2 + \sqrt{3})$	(d)	$7 - 4\sqrt{3}$
	(e)	$\sqrt{3} + \sqrt{2}$	(f)	$\sqrt{5} - \sqrt{3}$	(g)	$\frac{\sqrt{2}(\sqrt{5}-3)}{-4}$	(h)	$\frac{12-3\sqrt{2}}{7}$

Surds (2)-Problems

1.	(a)	$3\sqrt{5}$	(b)	$\sqrt{3}$	(c)	$6\sqrt{5}$	(d)	$2\sqrt{2}$
2.	(a)	10	(b)	-2	(c)	6	(d)	$4\sqrt{2}$
3.	(a)	$4\sqrt{3}$	(b)	8	(c)	$4\sqrt{15}$		
4.	(i)	2	(ii)	$2\sqrt{3}$				
5.	(a)	3	(b)	$\sqrt{11}$				
6.	$8\sqrt{2}$							
7.	(a)	$\sqrt{3}a$	(b)	$\sqrt{3}a^2$				
8.	Proof.	9.	$-1 + \sqrt{7}$	10.	Proof.			

Indices

1. (a) y^3 (b) p^4q^2 (c) a^3b^2 (d) 2^7 (e) p^{10} (f) a^{-1}
(g) $6y^3$ (h) $2ab^3$ (i) y^2 (j) $2r^5$ (k) d^7 (l) $3a^5$
(m) h^3 (n) $\frac{2}{e^6}$ (o) $2p^4$ (p) $2a^3$ (q) $\frac{1}{wp^2}$ (r) $\frac{a^2}{2b^2}$

2. (a) 3^{10} (b) $\frac{1}{4^6}$ (c) a^{21} (d) $\frac{1}{p^{12}}$ (e) d^{10} (f) a^2
(g) $y^{\frac{1}{2}}$ (h) $\frac{2}{p^2}$ (i) 3 (j) $k^{\frac{2}{15}}$ (k) a^3b^3 (l) x^4y^8
(m) $8m^9$ (n) $16x^4y^8$ (o) $3ab^2$

3. (a) $a^{\frac{1}{4}}$ (b) $p^{\frac{3}{2}}$ (c) $x^{\frac{5}{3}}$ (d) $r^{\frac{2}{5}}$ (e) $b^{\frac{3}{a}}$ (f) $h^{\frac{2}{3}}$

4. (a) $\sqrt[5]{p}$ (b) $\sqrt[4]{w^3}$ (c) $\sqrt{x}$ (d) $\frac{1}{\sqrt[4]{a^3}}$ (e) $\frac{1}{\sqrt[5]{y}}$ (f) $\frac{1}{\sqrt[3]{q^4}}$

5. (a) 5 (b) 2 (c) $\frac{1}{2}$ (d) $\frac{1}{2}$ (e) 1 (f) $\frac{1}{7}$
(g) 64 (h) 9 (i) $\frac{1}{16}$ (j) -2 (k) 16 (l) $\frac{1}{1000}$
(m) 2 (n) $\frac{1}{16}$ (o) $\frac{8}{27}$ (p) 32 (q) $\frac{16}{9}$ (r) $\frac{81}{16}$

6. (a) $x^{\frac{9}{2}} + x^{\frac{1}{2}}$ (b) $x - x^{\frac{3}{2}}$ (c) $x^{\frac{-3}{2}} + x^{-1}$ (d) $2x^5 + 2x^2$
(e) $x^{\frac{3}{2}} - 1$ (f) $x^4 + 2x + x^{-2}$ (g) $x^{\frac{-1}{2}} + 1$ (h) $x^2 + 2x^{\frac{1}{2}} + x^{-1}$
(i) $x^{-3} - x^{\frac{-5}{3}}$ (j) $x + 3x^{-1}$ (k) $x^{\frac{-3}{2}} - x^{-1}$ (l) $4x^{\frac{1}{2}} + 4x^{\frac{-1}{2}} + x^{\frac{-3}{2}}$

Exponential Graphs

1.

t	0	1	2	3	4	5	6
W	3	6	12	24	48	96	192

Student's graph

2.

P	0	1	2	3	4
F	80	40	20	10	5

Student's graph

3. (a) 10 (b) 3

4. (a) 4 (b) 5

5. (a) 50 (b) 2

Systems of Equations (Problems)

1. (a) $x = 2{\cdot}3, y = 3{\cdot}48$ (b) $a = 5{\cdot}4, b = 2{\cdot}6$
2. 5 hours
3. 61 racing and 21 cross country
4. 17
5. (a) $2u + 2a = 48$, $6u + 18a = 192$, 20, 4 (b) 400
6. (a) $u^2 + 40a = 100$, $u^2 + 180a = 324$, 6, 1·6 (b) 26

Mixed Exercise 1

1. (a) 7 (b) 2
2. (a) 70° (b) 17·1cm (c) 27·7cm^2
3. (a) the square of v and inversely as t. (b) 0·01, 8
4. (a) $12ab$
5. 7920m^3
6. $\dfrac{x+1}{6}$
7. 0·29, 1·71
8. $x = \dfrac{1+2y^2}{1-y^2}$

Mixed Exercise 2

1. (a) $x < 4$ (b) $(5x + 1)(x - 4)$, (i) -0·2, 4 (ii) $\dfrac{x}{5x+1}$
   (c) (4·5,-3) (d) -2
2. 8, 2
3. (a) $v = \dfrac{fu}{(u-f)}$ (b) 52·08
4. (a) 5·9 (b) 32° (c) 45.2
5. $61013\frac{1}{3}$ cm^3
6. 26·6°, 153·4°

Mixed Exercise 3

1. $-2, \frac{1}{4}$

2. (a) $3x + 1$ (b) $4x^2 + 8x + 4, 3x^2 + 10x + 3$ (c) 5

3. (a) 120° (b) 13m

4. $1{\cdot}6 \times 10^{11}$

5. (a) $4a + \frac{1}{2}b = 30$, $16a + \frac{1}{4}b = 50$, 2·5, 40 (b) £254

6. (i) 2·90cm (ii) 25·4cm^2 (c) 7·35cm^3

7. 17km/h

Mixed Exercise 4

1. (i) 1 (ii) 2, 6

2. 10cm

3. 0·288cm

4. 88cm

5. -0·24, 0·84

6. 8 **more** men

7. 7·41cm^2

8. 1·35litres

9. (a) $\dfrac{3x}{x+2}$ (b) $\dfrac{2a+3}{a+4}$